POLITECNICO
MILANO 1863

Elettromagnetismo

Elettricità. Corrente. Magnetismo

Maurizio Zani

Sommario

Elettromagnetismo

http://www.mauriziozani.it/wp/?p=1128

Elettromagnetismo

Circuiti elettrici variabili

Situazione stazionaria

- campi e correnti costanti nel tempo
- equazioni di Maxwell stazionarie
- leggi di Kirchhoff

Situazione non stazionaria

- campi e correnti variabili
- equazioni di Maxwell complete

Situazione quasi stazionaria

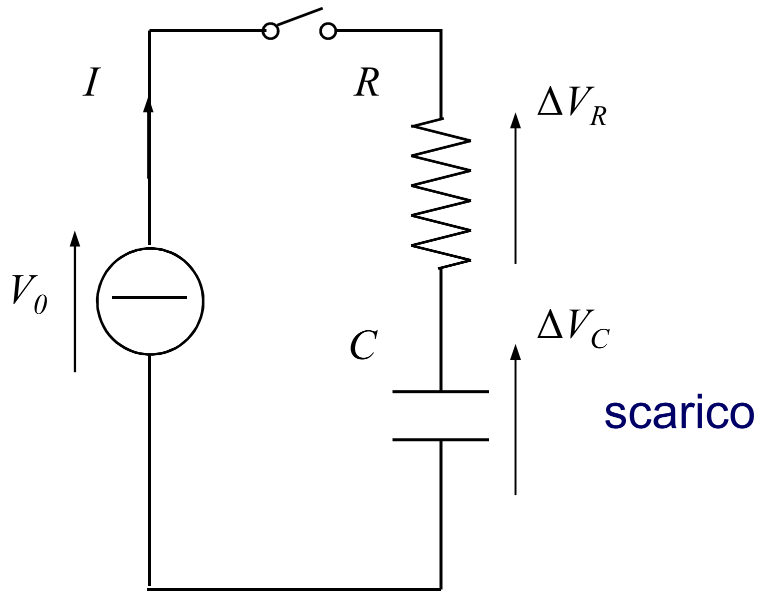
- campi e correnti lentamente variabili
- leggi di Kirchhoff
- circuiti in transitorio
  - processi di carica/scarica
- circuiti oscillanti
  - ideali e reali

Circuiti in transitorio: carica del condensatore

circuito RC

$$V_0 = \Delta V_R + \Delta V_C = RI + \frac{q}{C} = R\frac{dq}{dt} + \frac{q}{C}$$

legge di Kirchhoff

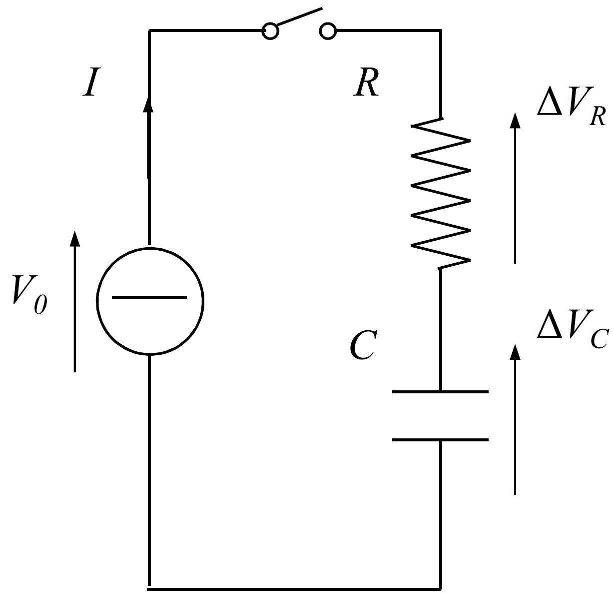


$$\int_0^q \frac{dq}{q - CV_0} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q - CV_0}{-CV_0}\right) = -\frac{1}{RC}t$$

$$q = CV_0\left(1 - e^{-\frac{t}{RC}}\right)$$

Circuiti in transitorio: carica del condensatore

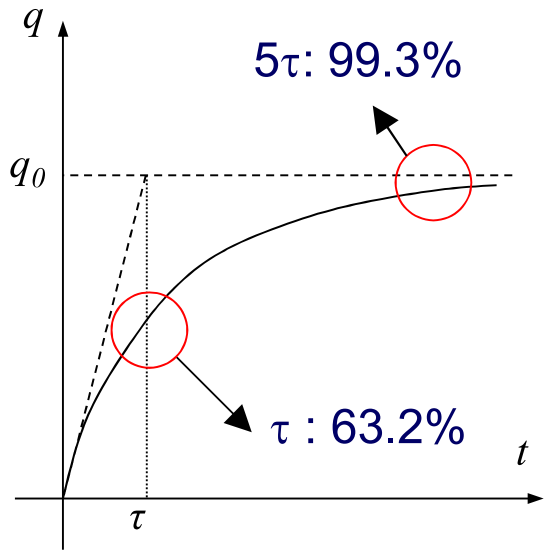


$$q = CV_0\left(1 - e^{-\frac{t}{RC}}\right) = q_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

$q_0 = CV_0$ carica finale

$\tau = RC$

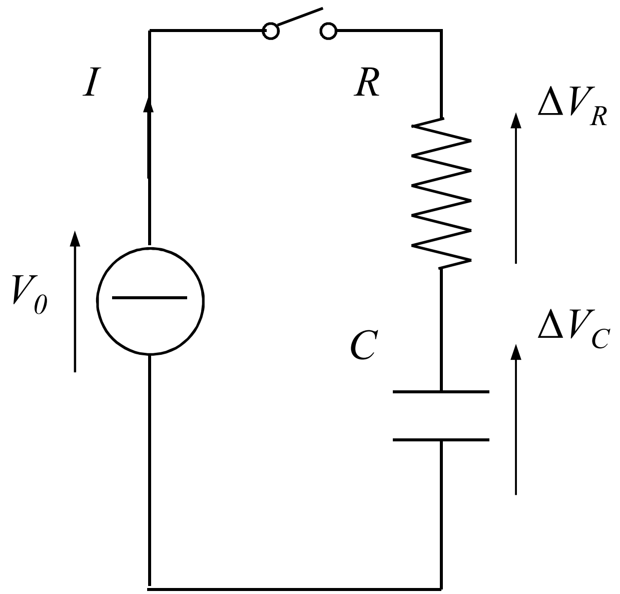
costante di tempo



$$\left.\frac{dq}{dt}\right|_{t=0} = \frac{q_0}{\tau}$$

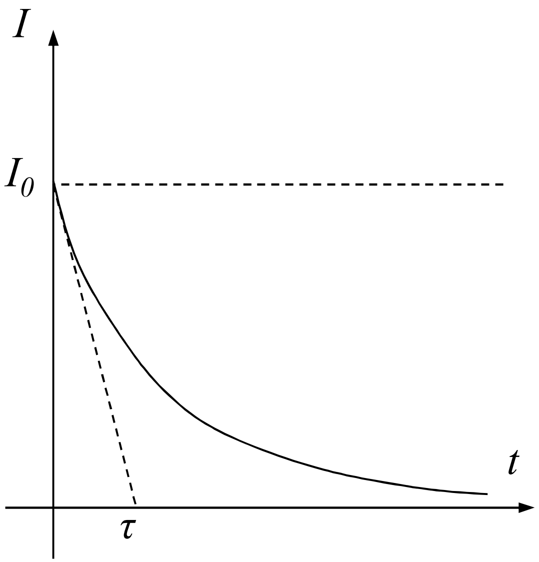
intercetta

Circuiti in transitorio: carica del condensatore



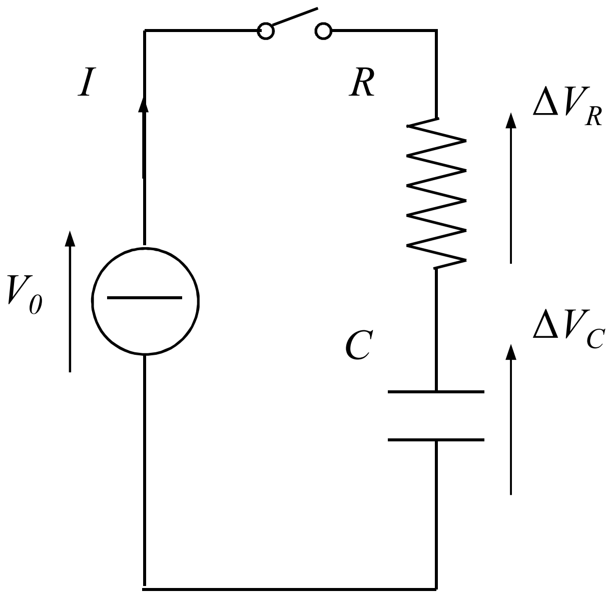
$$I = \frac{\mathrm{d}q}{\mathrm{d}t} = \frac{V_0}{R}\mathrm{e}^{-\frac{t}{\tau}} = I_0\mathrm{e}^{-\frac{t}{\tau}}$$

$$I_0 = \frac{V_0}{R}$$ corrente iniziale

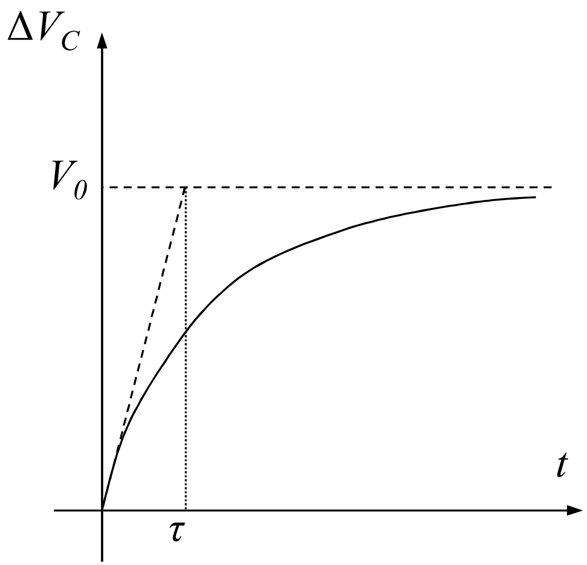


Circuiti in transitorio: carica del condensatore

$$\Delta V_C = \frac{q}{C} = \frac{q_0}{C}\left(1 - e^{-\frac{t}{\tau}}\right) = V_0\left(1 - e^{-\frac{t}{\tau}}\right)$$



$$V_0 = \frac{q_0}{C}$$ tensione finale

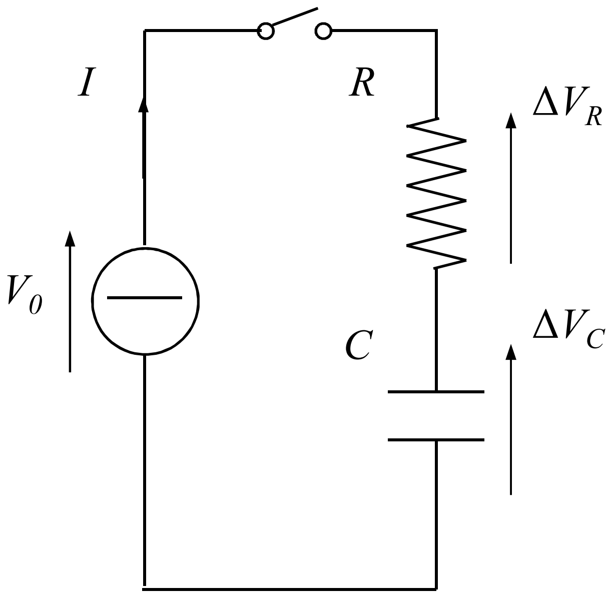


$$W_C = \frac{1}{2} C V_0^2$$

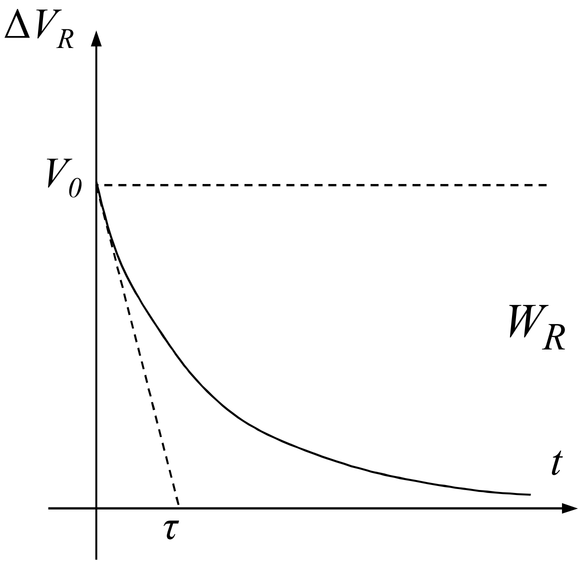
energia accumulata

Circuiti in transitorio: carica del condensatore

$$\Delta V_R = RI = RI_0 e^{-\frac{t}{\tau}} = R\frac{V_0}{R} e^{-\frac{t}{\tau}} = V_0 e^{-\frac{t}{\tau}}$$



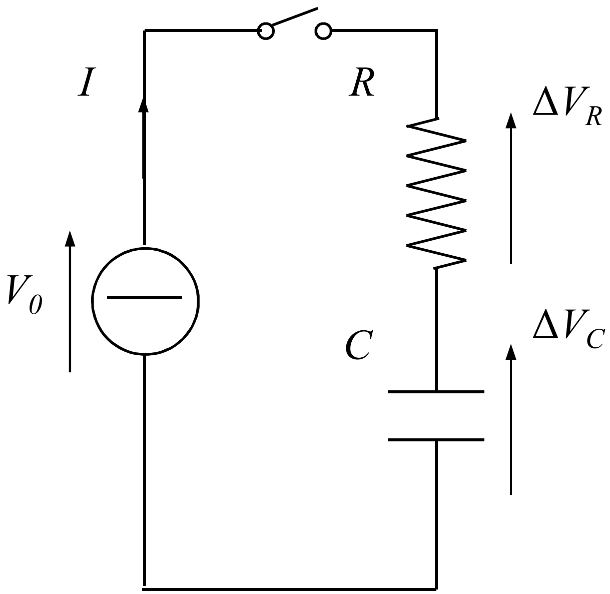
$V_0 = RI_0$ tensione iniziale

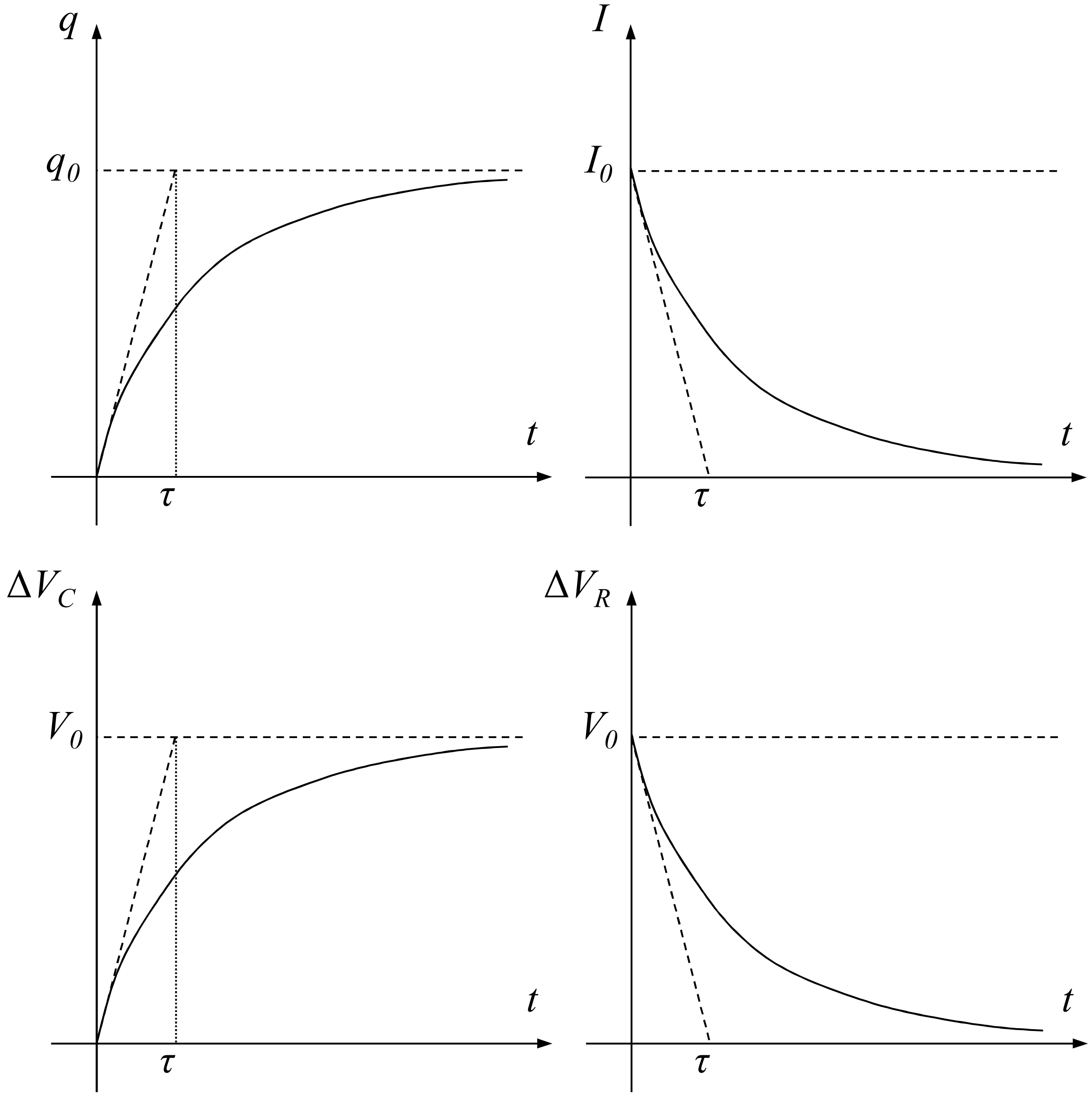


$$W_R = \int_0^{\infty} RI^2 \mathrm{d}t = \frac{1}{2} C V_0^2 = W_C$$

energia dissipata

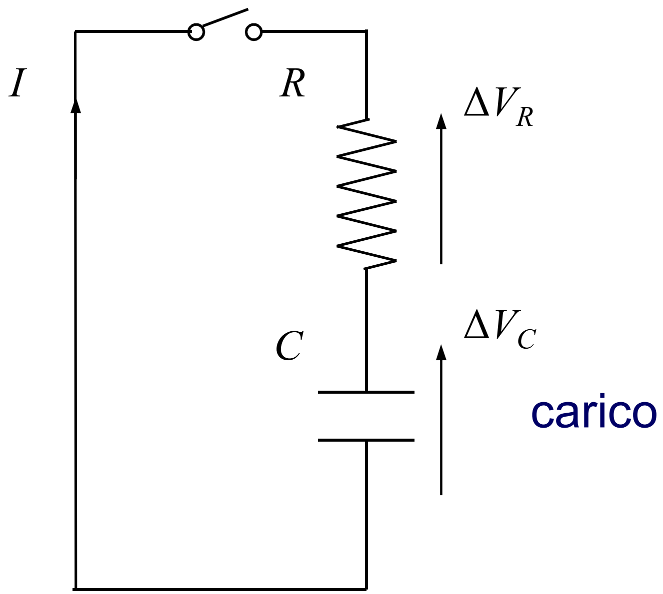
Circuiti in transitorio: carica del condensatore





Circuiti in transitorio: scarica del condensatore

circuito RC



$$\Delta V_R + \Delta V_C = RI + \frac{q}{C} = R\frac{\mathrm{d}q}{\mathrm{d}t} + \frac{q}{C} = 0$$

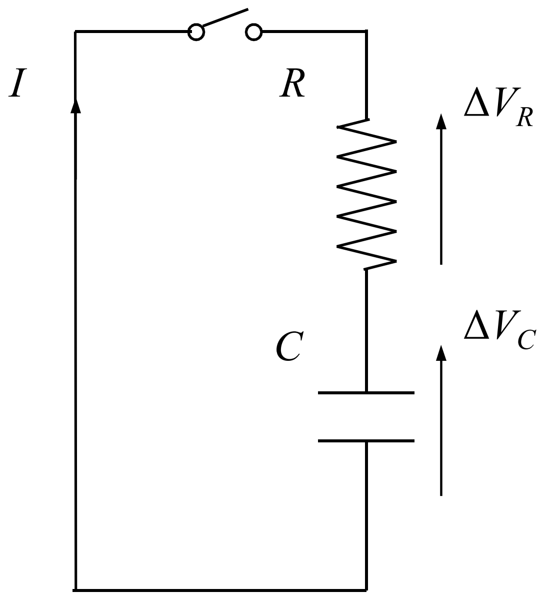
legge di Kirchhoff

$$\int_{q_0}^{q} \frac{\mathrm{d}q}{q} = -\frac{1}{RC}\int_0^t \mathrm{d}t$$

$$\ln\left(\frac{q}{q_0}\right) = -\frac{1}{RC}t$$

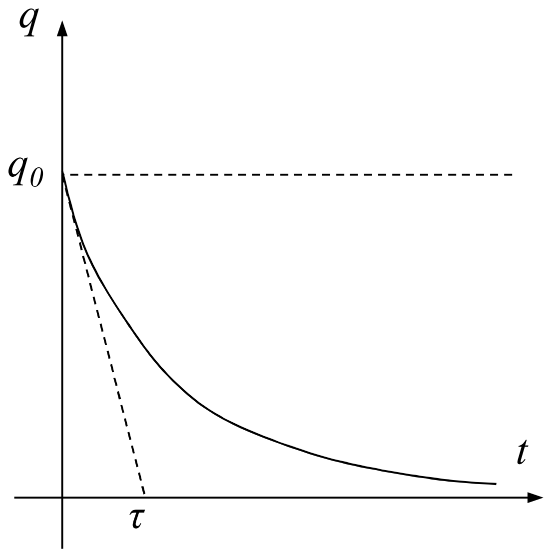
$$q = q_0 \mathrm{e}^{-\frac{t}{RC}}$$

Circuiti in transitorio: scarica del condensatore



$$q = q_0 e^{-\frac{t}{RC}} = q_0 e^{-\frac{t}{\tau}}$$

$q_0 = CV_0$ carica iniziale



$\tau = RC$

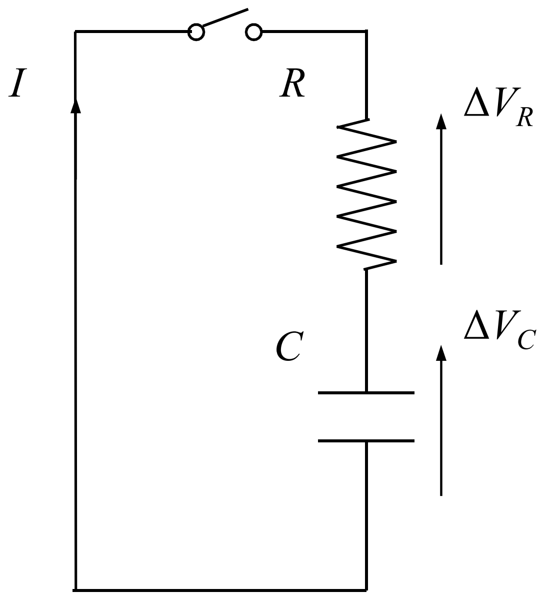
costante di tempo

$$\left.\frac{dq}{dt}\right|_{t=0} = \frac{q_0}{\tau}$$

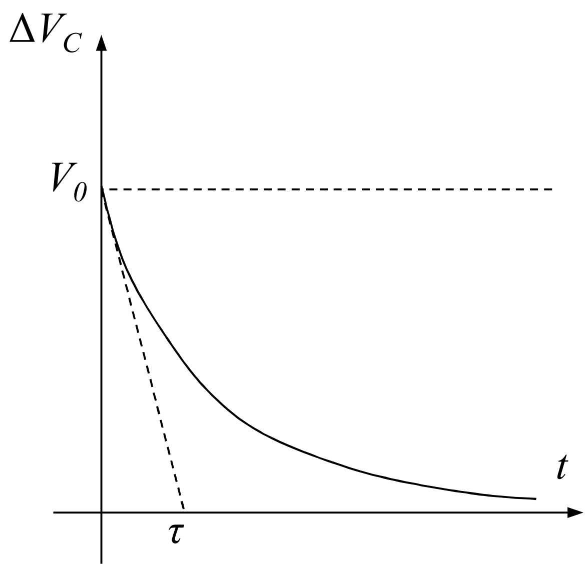
intercetta

Circuiti in transitorio: scarica del condensatore

$$\Delta V_C = \frac{q}{C} = \frac{q_0}{C} e^{-\frac{t}{\tau}} = V_0 e^{-\frac{t}{\tau}}$$



$$V_0 = \frac{q_0}{C}$$ tensione iniziale



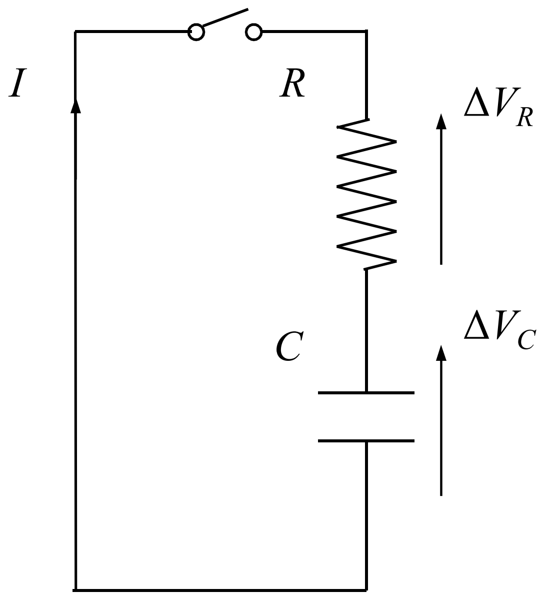
$$W_C = \frac{1}{2} C V_0^2$$

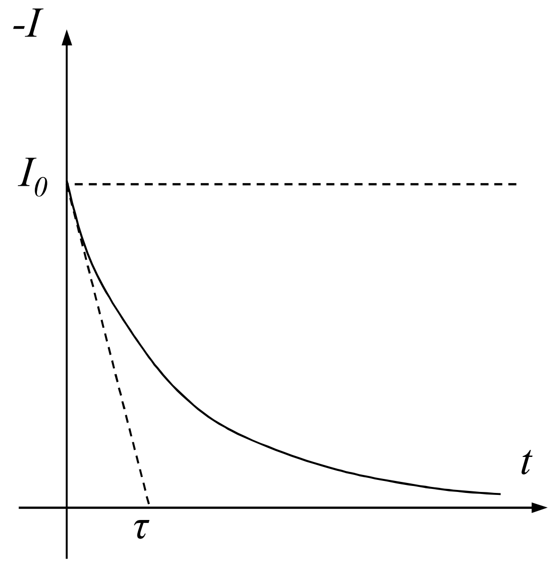
energia rilasciata

Circuiti in transitorio: scarica del condensatore

$$I = \frac{\mathrm{d}q}{\mathrm{d}t} = -q_0 \frac{1}{\tau} \mathrm{e}^{-\frac{t}{\tau}} = -\frac{V_0}{R} \mathrm{e}^{-\frac{t}{\tau}} = -I_0 \mathrm{e}^{-\frac{t}{\tau}}$$

$$I_0 = \frac{V_0}{R}$$ corrente iniziale

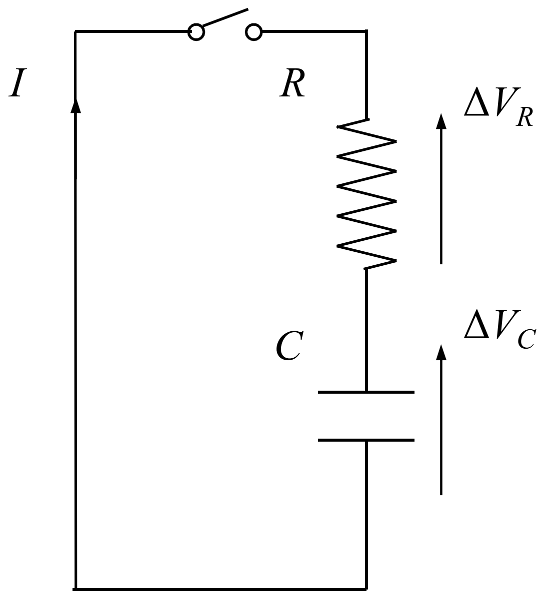


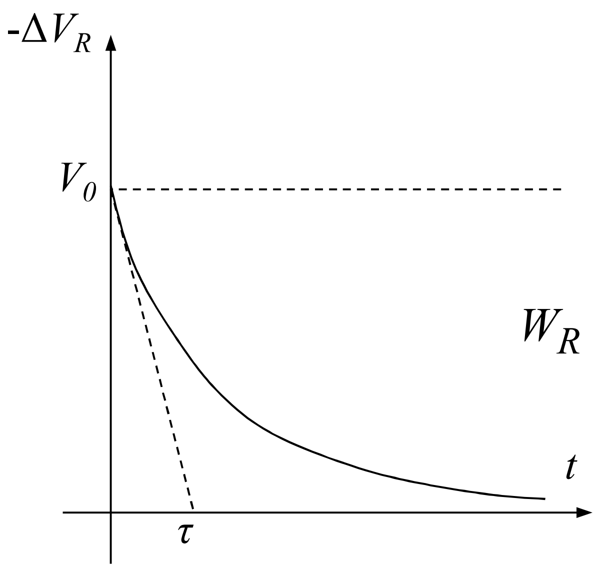


Circuiti in transitorio: scarica del condensatore

$$\Delta V_R = RI = -RI_0 e^{-\frac{t}{\tau}} = -V_0 e^{-\frac{t}{\tau}}$$

$$V_0 = RI_0$$ tensione iniziale

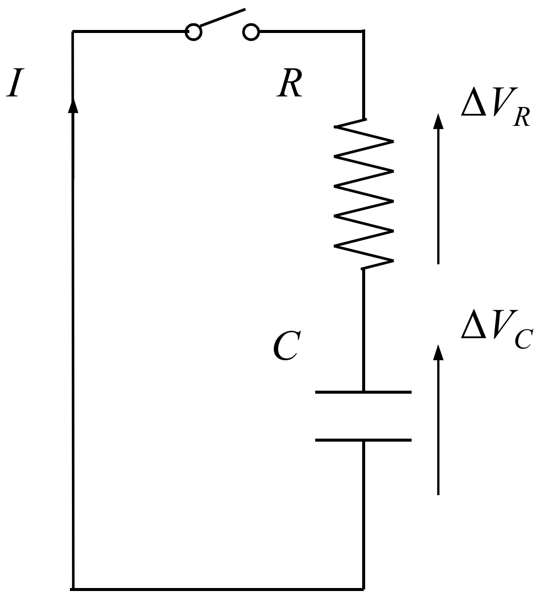


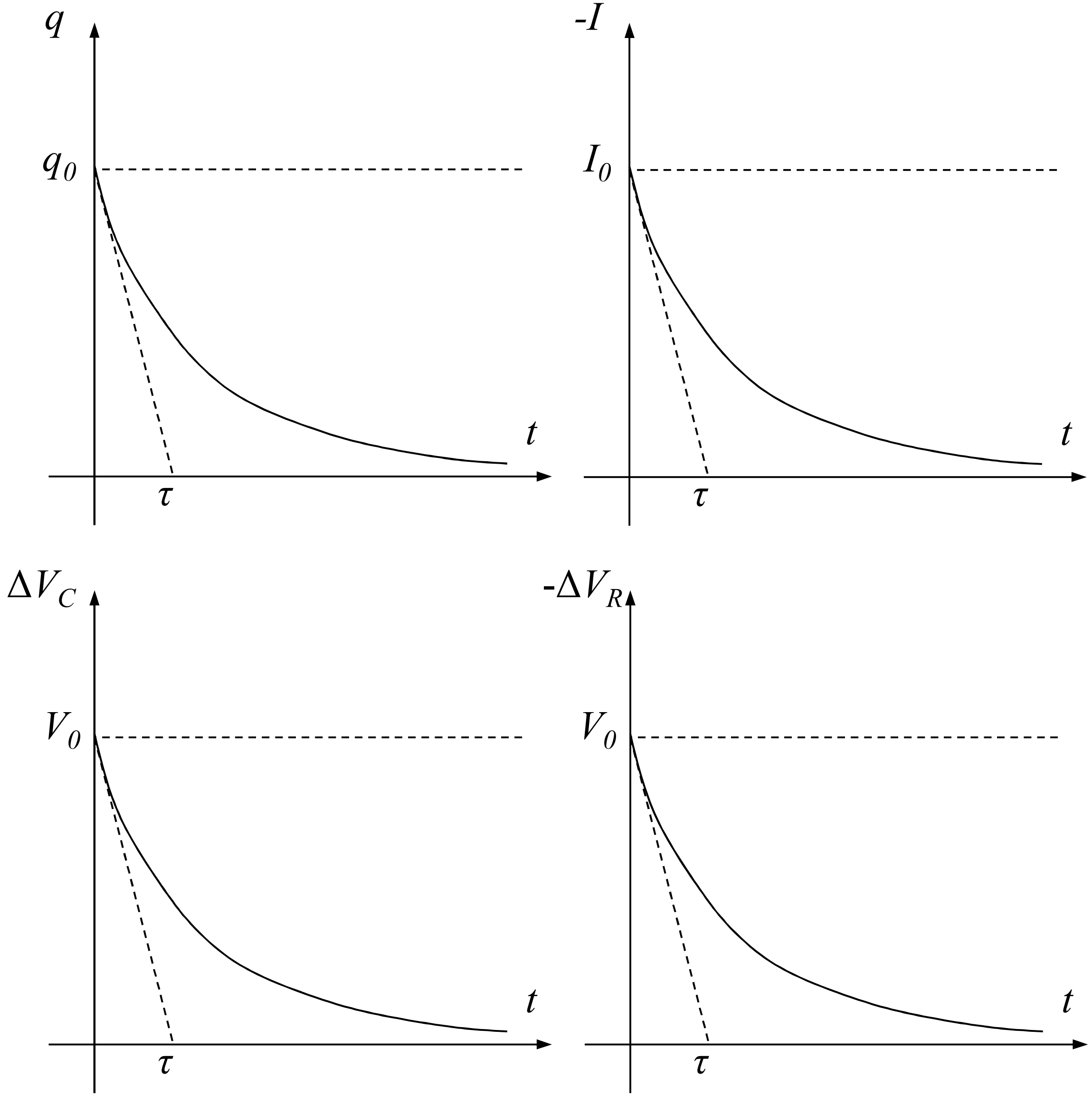


$$W_R = \int_0^{\infty} RI^2 \mathrm{d}t = \frac{1}{2} C V_0^2 = W_C$$

energia dissipata

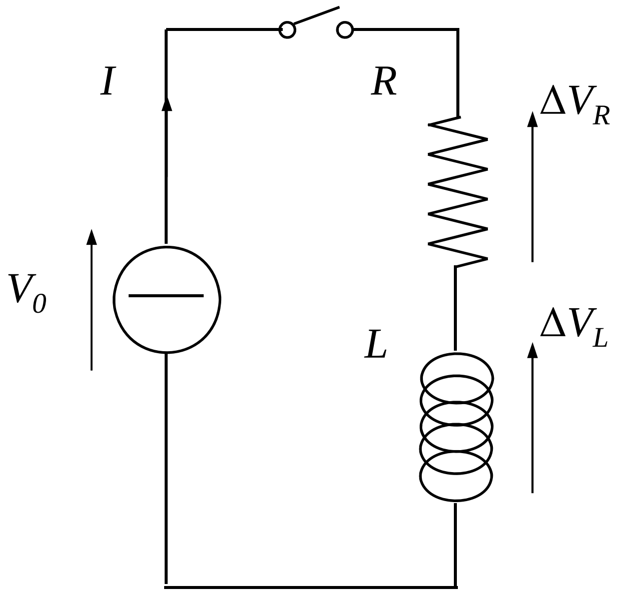
Circuiti in transitorio: scarica del condensatore





Circuiti in transitorio: carica dell'induttore

circuito RL



$$V_0 = \Delta V_R + \Delta V_L = RI + L\frac{dI}{dt}$$

legge di Kirchhoff

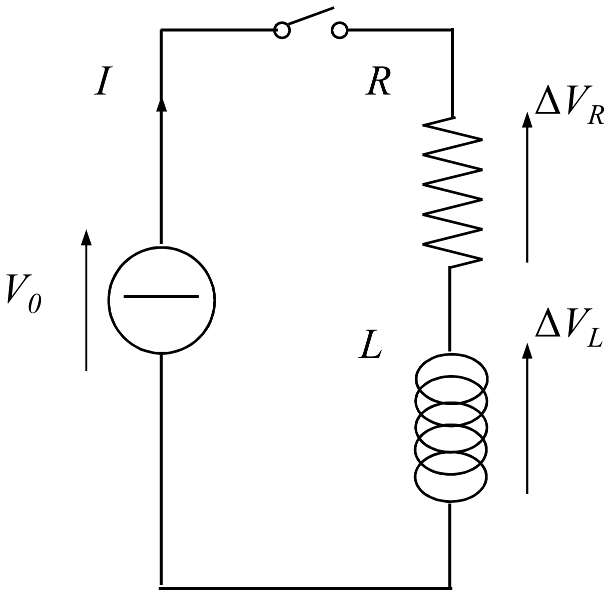
$$\int_0^I \frac{dI}{V_0 - RI} = \frac{1}{L}\int_0^t dt$$

$$-\frac{1}{R}\ln\left(\frac{V_0 - RI}{V_0}\right) = \frac{1}{L}t$$

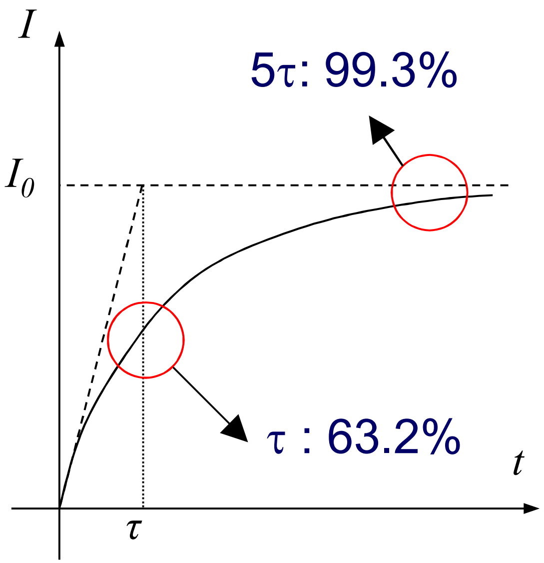
$$I = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

Circuiti in transitorio: carica dell'induttore

$$I = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right) = I_0\left(1 - e^{-\frac{t}{\tau}}\right)$$



$$I_0 = \frac{V_0}{R}$$ corrente finale



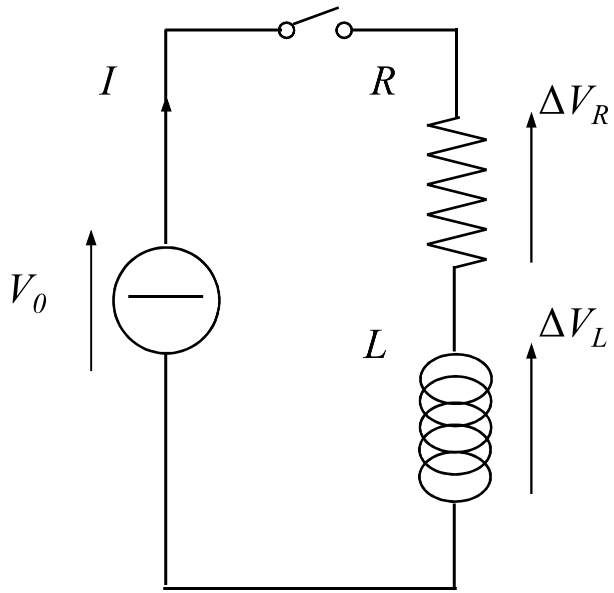
$$\tau = \frac{L}{R}$$

costante di tempo

$$\left.\frac{dI}{dt}\right|_{t=0} = \frac{I_0}{\tau}$$

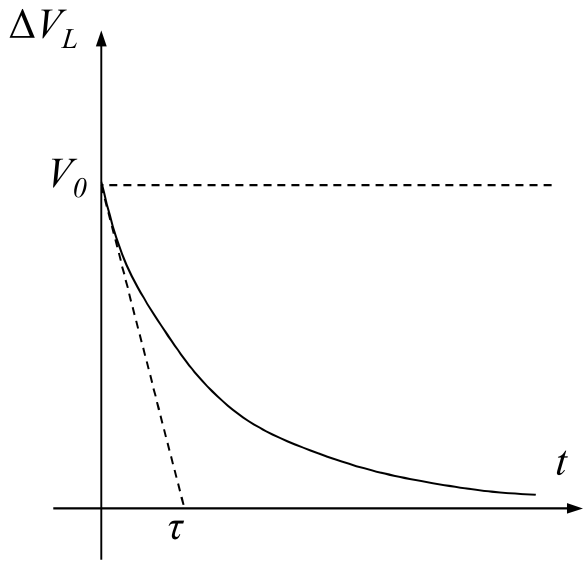
intercetta

Circuiti in transitorio: carica dell'induttore



$$\Delta V_L = L\frac{\mathrm{d}I}{\mathrm{d}t} = LI_0\frac{1}{\tau}e^{-\frac{t}{\tau}} = V_0 e^{-\frac{t}{\tau}}$$

$V_0 = RI_0$ tensione iniziale

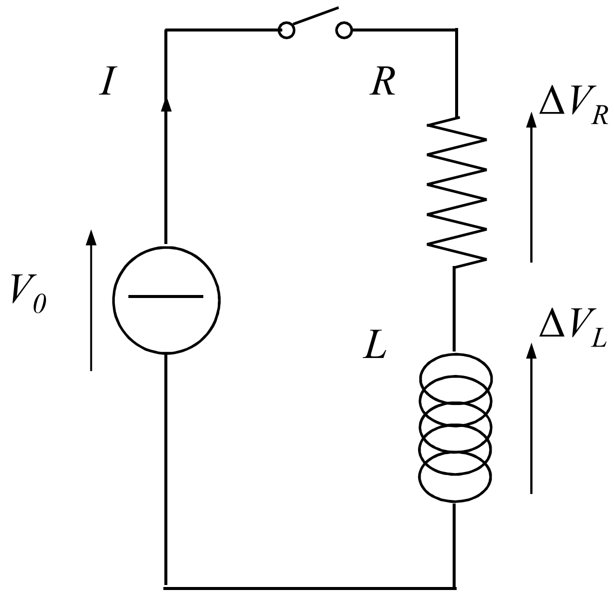


$$W_L = \frac{1}{2}LI_0^2$$

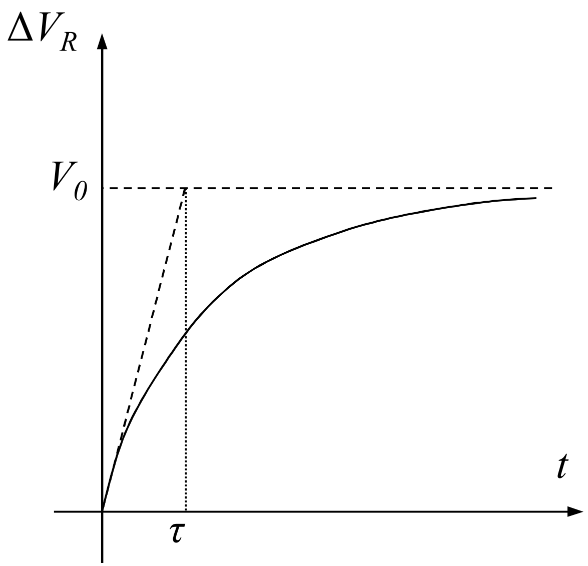
energia accumulata

Circuiti in transitorio: carica dell'induttore

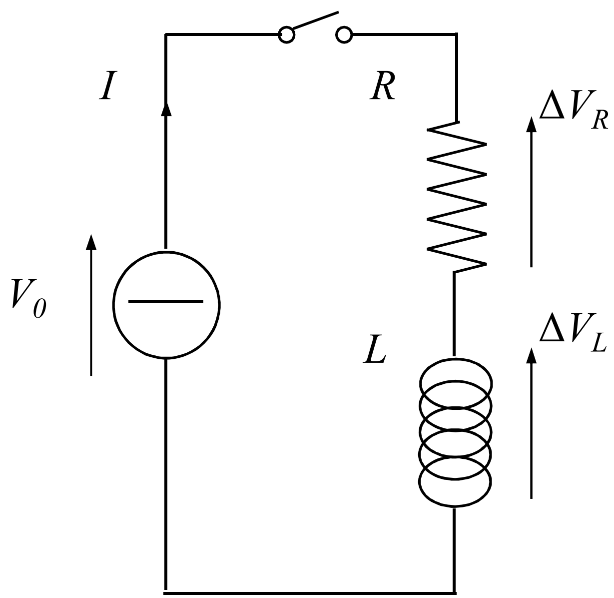
$$\Delta V_R = RI = RI_0\left(1 - e^{-\frac{t}{\tau}}\right) = V_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

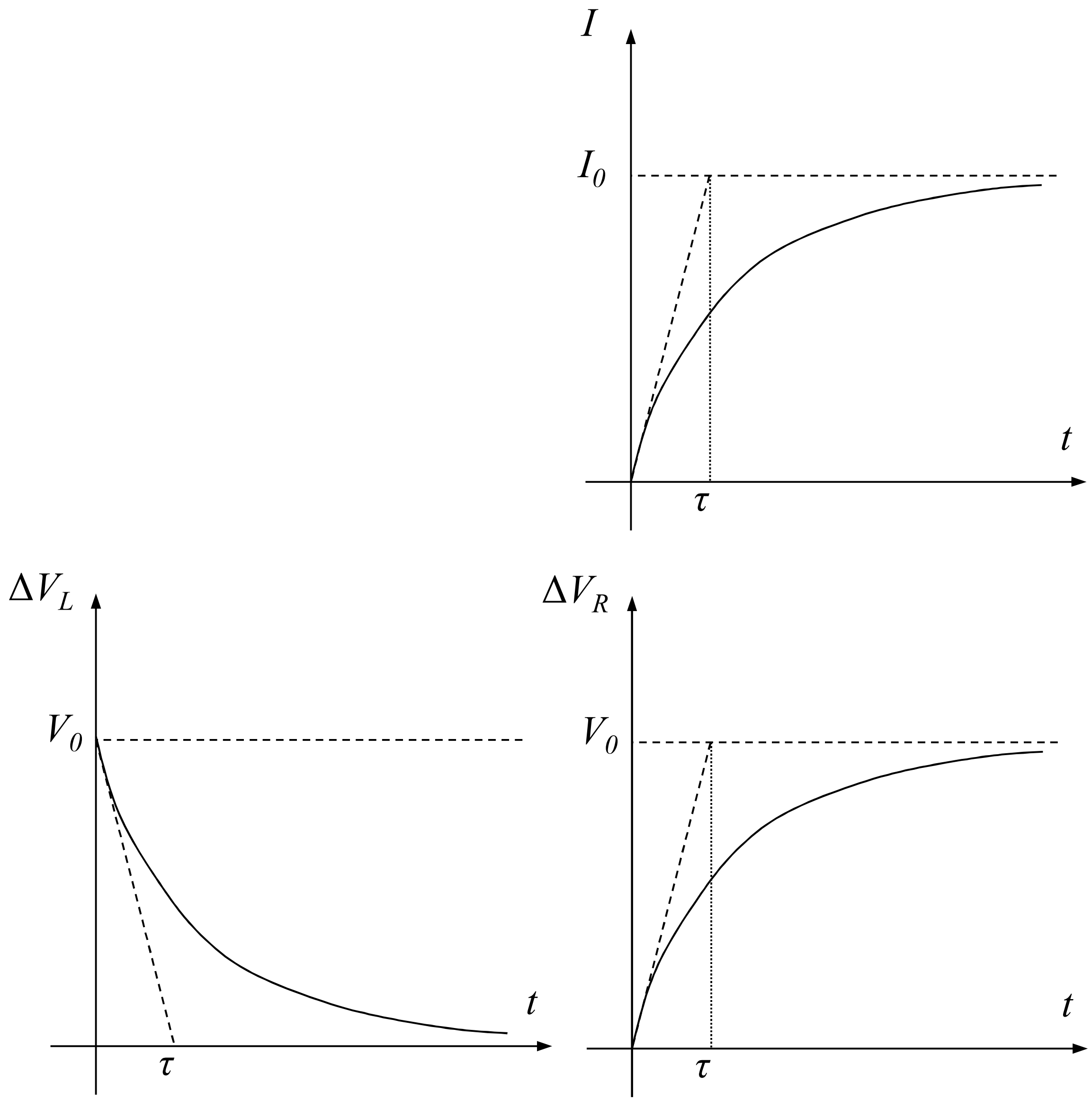


$V_0 = RI_0$ tensione finale



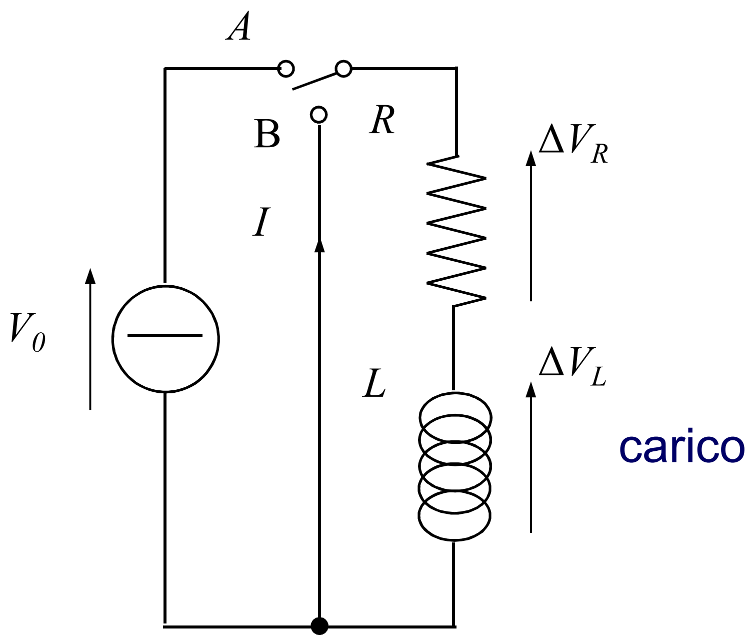
Circuiti in transitorio: carica dell'induttore





Circuiti in transitorio: scarica dell'induttore

circuito RL



$$\Delta V_R + \Delta V_L = RI + L\frac{dI}{dt} = 0$$

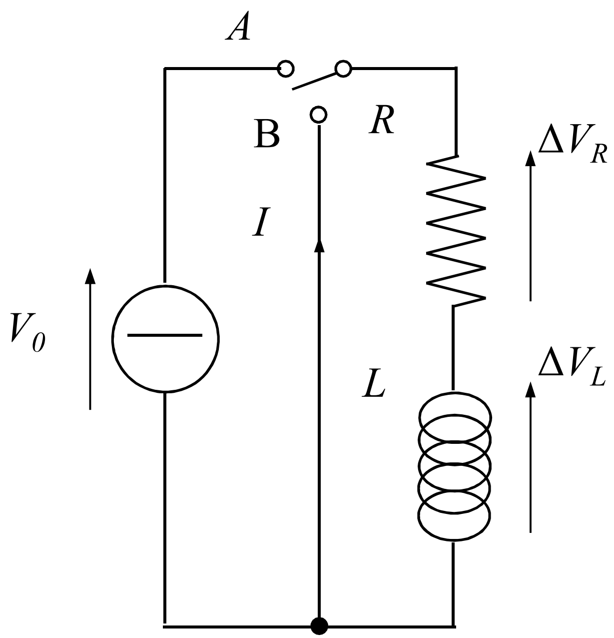
legge di Kirchhoff

$$\int_{I_0}^{I} \frac{dI}{I} = -\frac{R}{L}\int_0^t dt$$

$$\ln\left(\frac{I}{I_0}\right) = -\frac{R}{L}t$$

$$I = I_0 e^{-\frac{R}{L}t}$$

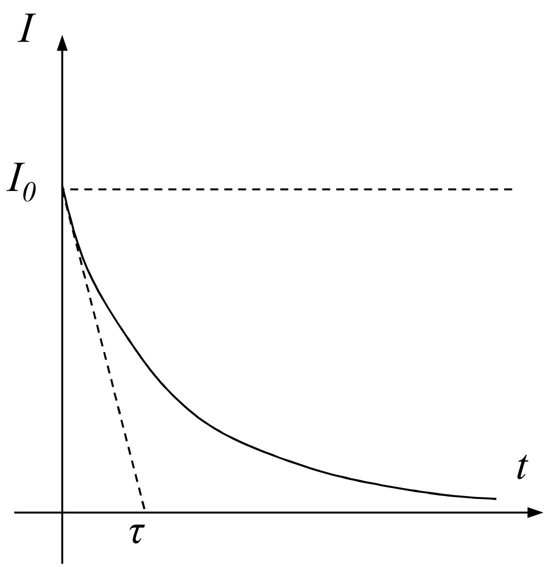
Circuiti in transitorio: scarica dell'induttore



$$I = I_0 e^{-\frac{R}{L}t} = I_0 e^{-\frac{t}{\tau}}$$

$$I_0 = \frac{V_0}{R}$$ corrente iniziale

extracorrente di apertura



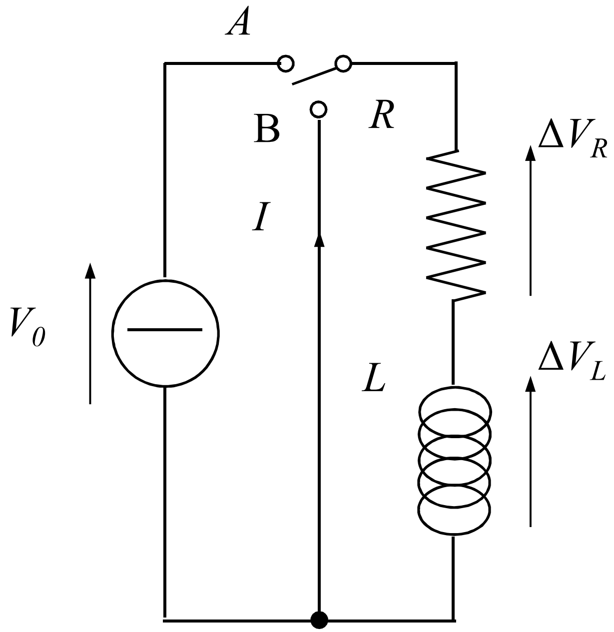
$$\tau = \frac{L}{R}$$

costante di tempo

$$\left.\frac{dI}{dt}\right|_{t=0} = \frac{I_0}{\tau}$$

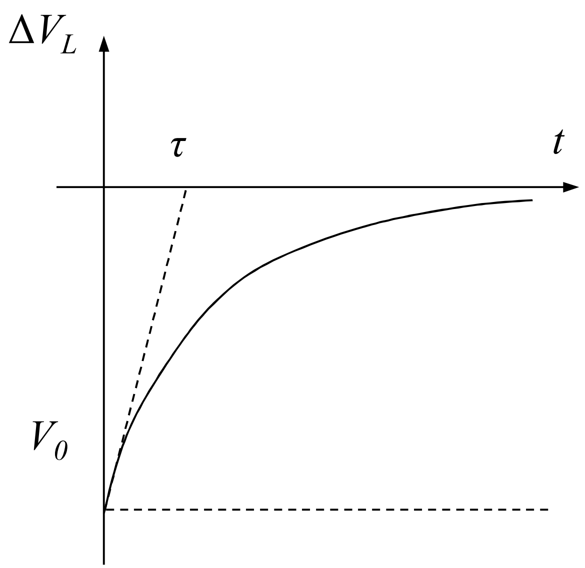
intercetta

Circuiti in transitorio: scarica dell'induttore



$$\Delta V_L = L\frac{dI}{dt} = -LI_0\frac{1}{\tau}e^{-\frac{t}{\tau}} = -V_0 e^{-\frac{t}{\tau}}$$

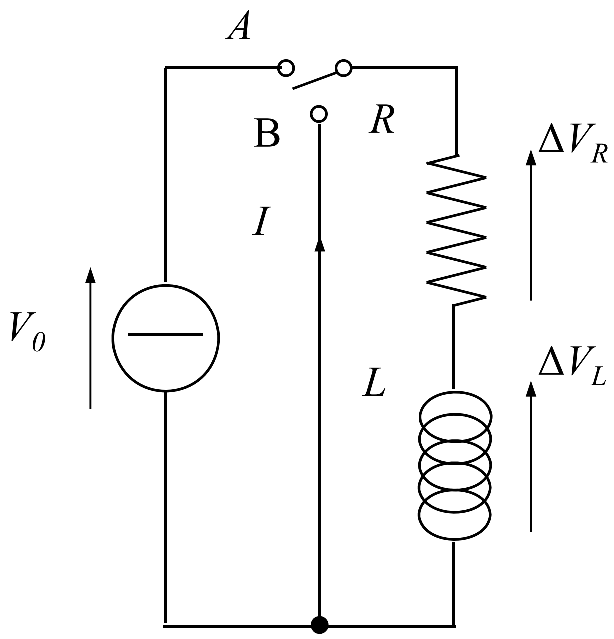
$$-V_0 = RI_0$$ tensione iniziale



$$W_L = \frac{1}{2}LI_0^2$$

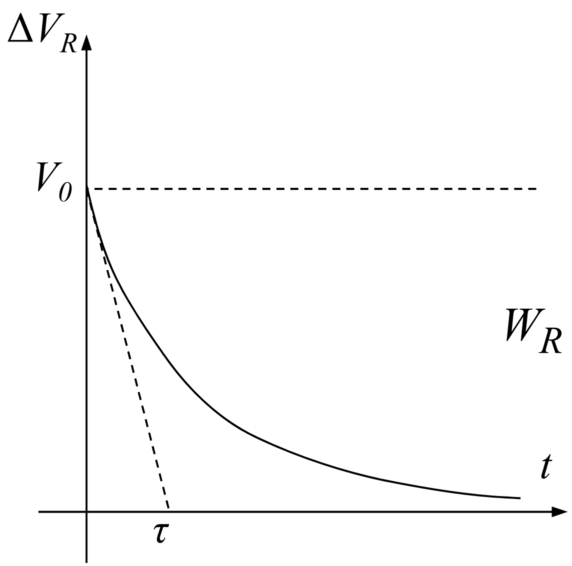
energia rilasciata

Circuiti in transitorio: scarica dell'induttore



$$\Delta V_R = RI = RI_0 e^{-\frac{t}{\tau}} = V_0 e^{-\frac{t}{\tau}}$$

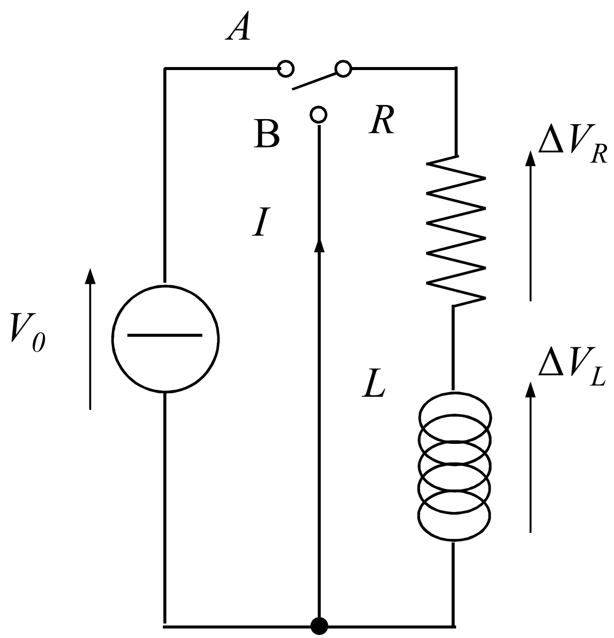
$$V_0 = RI_0$$ tensione iniziale

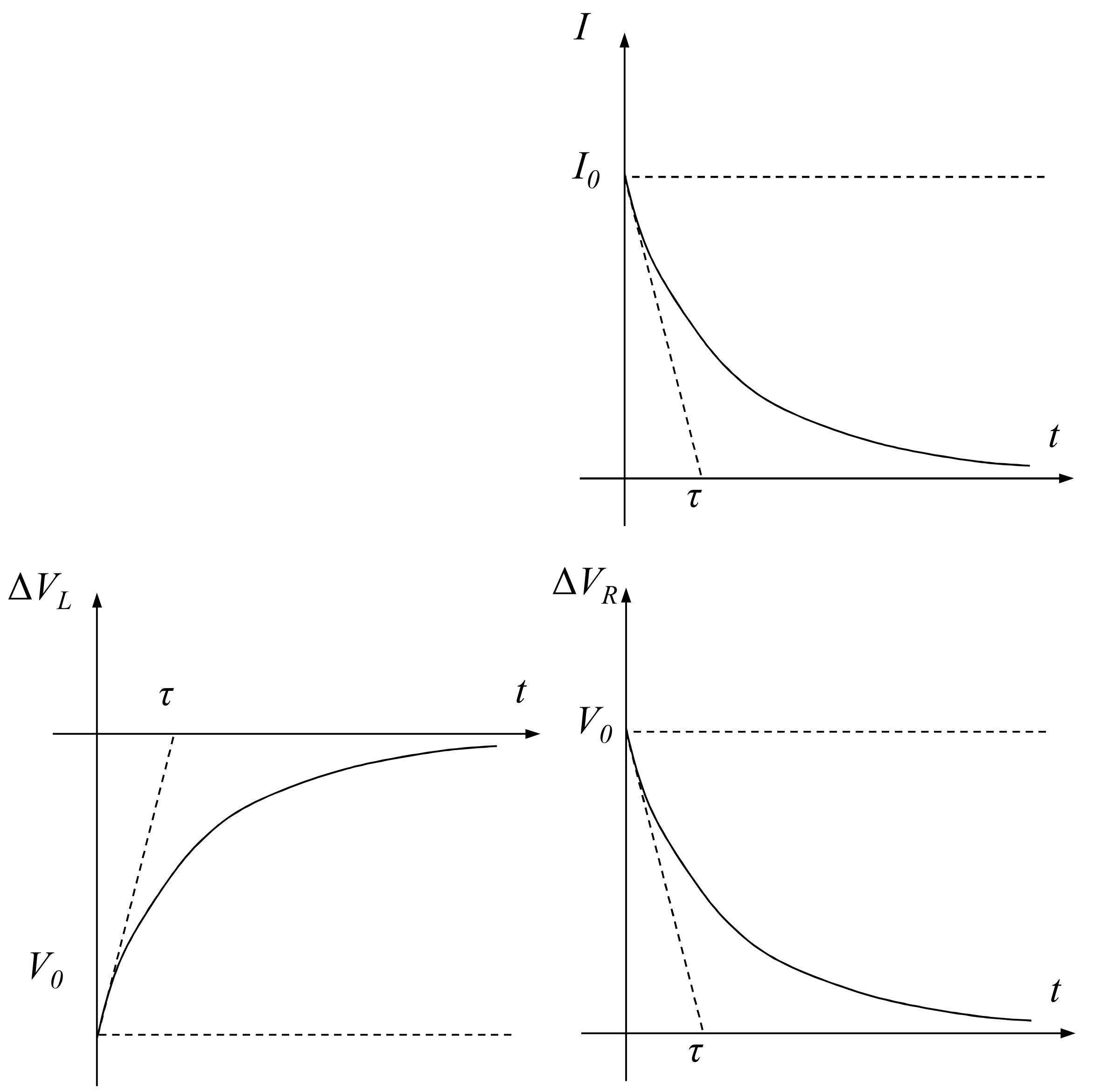


$$W_R = \int_0^\infty RI^2 dt = \frac{1}{2} LI_0^2 = W_L$$

energia dissipata

Circuiti in transitorio: scarica dell'induttore

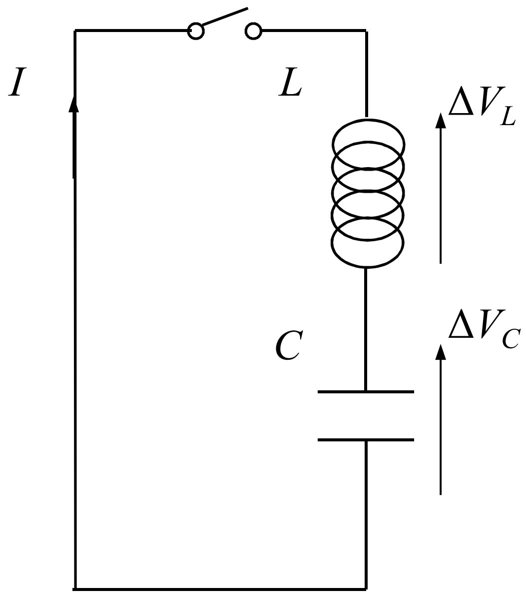




Circuiti oscillanti: oscillatore ideale

circuito LC

$$\Delta V_L + \Delta V_C = L\frac{\mathrm{d}I}{\mathrm{d}t} + \frac{q}{C} = L\frac{\mathrm{d}^2 q}{\mathrm{d}t^2} + \frac{q}{C} = 0$$



$$\frac{\mathrm{d}^2 q}{\mathrm{d}t^2} + \frac{1}{LC}q = 0$$

$$q = q_0 \sin\left(\omega_0 t - \varphi\right)$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

pulsazione propria

Circuiti oscillanti: oscillatore reale smorzato

circuito RLC