POLITECNICO
MILANO 1863

Elettromagnetismo

Elettricità. Corrente. Magnetismo

Maurizio Zani

Sommario

Elettromagnetismo

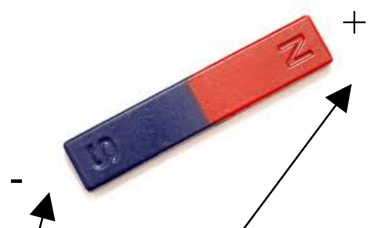
http://www.mauriziozani.it/wp/?p=1128

Elettromagnetismo

Elettrostatica
Materiali conduttori
Condensatori
Materiali dielettrici
Corrente elettrica
Resistori
Circuiti elettrici continui
Magnetostatica
Induzione elettromagnetica
Induttori
Materiali magnetici
Circuiti elettrici variabili
Elettromagnetismo

Magnetizzazione
Forza magnetica
Campo magnetico
Teorema di Gauss
Teorema di Ampère
Dipolo magnetico
Formulazione differenziale

Magnetizzazione



il materiale si magnetizza?

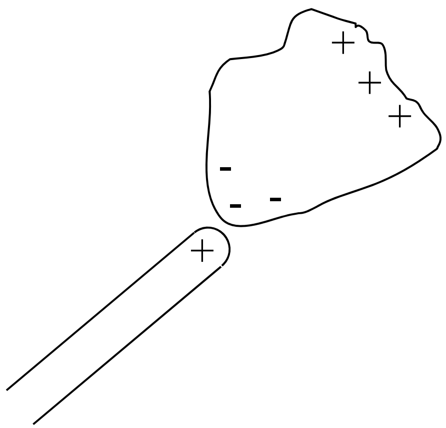
- no: **materiale "amagnetico"**
  - come rame e alluminio
- sì: **materiale magnetico**
  - come il ferro

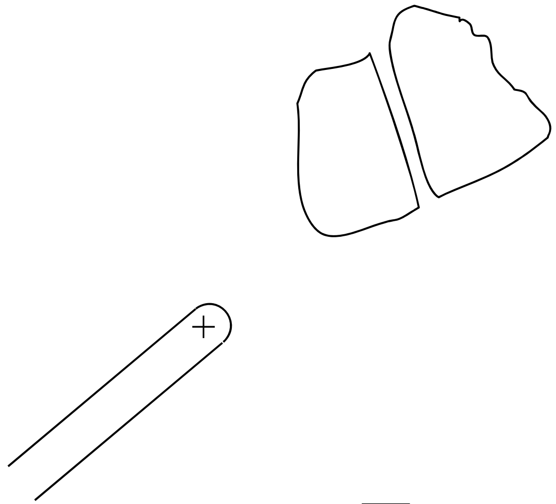
magnete permanente

- polo (p)
  - due tipi (S e N), vale algebra
- interazione
  - tipo diverso: attrazione
  - stesso tipo: repulsione

Magnetizzazione

materiali magnetici





prossimità (senza contatto)

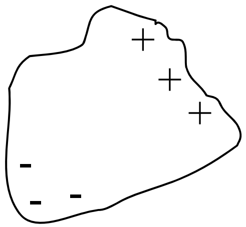
magnetizzazione
localizzata e temporanea

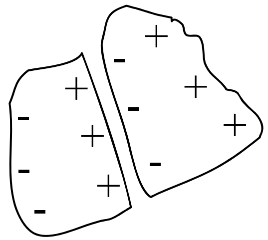
prossimità e taglio

nessuna
magnetizzazione

Magnetizzazione

magneti permanenti



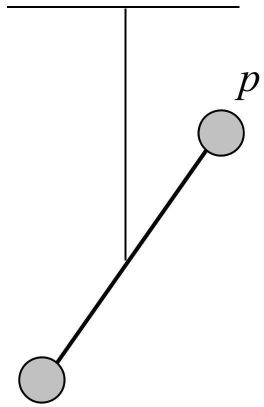


taglio

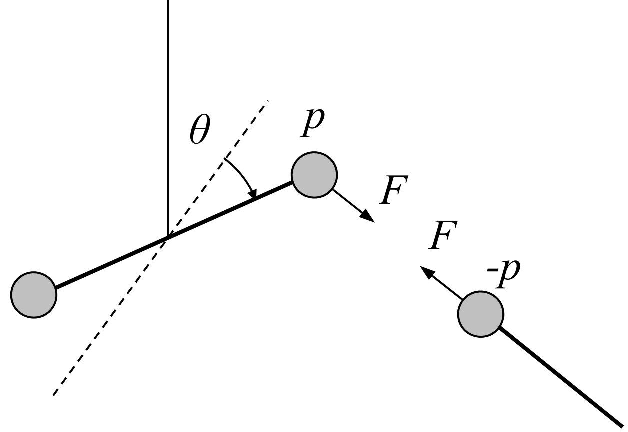
magnetizzazione permanente

magnetizzazione permanente in ogni frazione

Magnetizzazione: forza magnetica



bilancia di torsione



costante magnetica

$$\vec{F}_m = k_m \frac{p_1 p_2}{r^2} \vec{u}_r$$

$$k_m = \frac{\mu_0}{4\pi} = 10^{-7}\ \mathrm{N/A^2}$$

forza magnetica (di Lorentz)

$$\mu_0 = 4\pi \cdot 10^{-7}\ \mathrm{N/A^2}$$

forza fondamentale

permeabilità magnetica

Magnetizzazione: forza magnetica

$$c = 299\ 792\ 458 \text{ m/s}$$

$$\sqrt{\frac{k_e}{k_m}} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c$$

velocità della luce

$$k_m = \frac{\mu_0}{4\pi}$$

$$k_e = \frac{1}{4\pi\varepsilon_0}$$

$$\vec{F}_m = k_m \frac{p_1 p_2}{r^2} \vec{u}_r$$

forza magnetica (di Lorentz)

$$\vec{F}_e = k_e \frac{q_1 q_2}{r^2} \vec{u}_r$$

forza elettrica (di Coulomb)

Magnetizzazione: campo magnetico

$$\vec{F}_m = \frac{\mu_0}{4\pi}\frac{p_1 p_2}{r^2}\vec{u}_r$$

$$[B] = \frac{[F]}{[p]} = \frac{\mathrm{N}}{\mathrm{Am}} = \mathrm{T}$$

tesla

$$\vec{F}_m = \frac{\mu_0}{4\pi}\frac{p_1 p_2}{r^2}\vec{u}_r = p_1\left(\frac{\mu_0}{4\pi}\frac{p_2}{r^2}\vec{u}_r\right) = p_1\vec{B}_2$$

oggetto

effetto

causa

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{p_2}{r^2}\vec{u}_r$$

campo magnetico

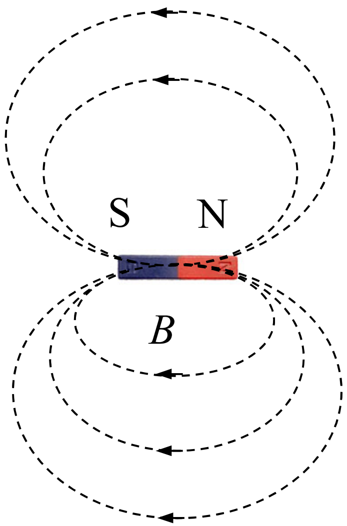
$$\vec{F}_1 = \sum \vec{F}_i = \sum p_1\vec{B} = p_1\sum \vec{B}_i = p_1\vec{B}$$

$$\begin{cases} \vec{B} = \sum \vec{B}_i \\ \vec{B} = \int \mathrm{d}\vec{B} \end{cases}$$

vale la sovrapposizione degli effetti

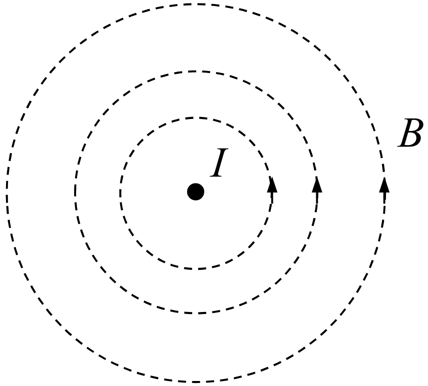
Magnetizzazione: campo magnetico

magnete



orientazione della
limatura di ferro

corrente



Linee di flusso

- linee orientate, tangenti (<u>direzione</u>) e concordi (<u>verso</u>) al campo
- si addensano dove il campo è più intenso
- non si incrociano mai
- partono (<u>sorgente</u>) e terminano (<u>pozzo</u>) sui poli o all'infinito

Forza magnetica

$$\vec{F}_m = q\vec{v} \times \vec{B}$$

forza magnetica (di Lorentz)

$$[B] = \frac{[F]}{[q][v]} = \frac{\mathrm{N}}{\mathrm{C\,m/s}} = \frac{\mathrm{N}}{\mathrm{Am}} = \mathrm{T}$$

tesla

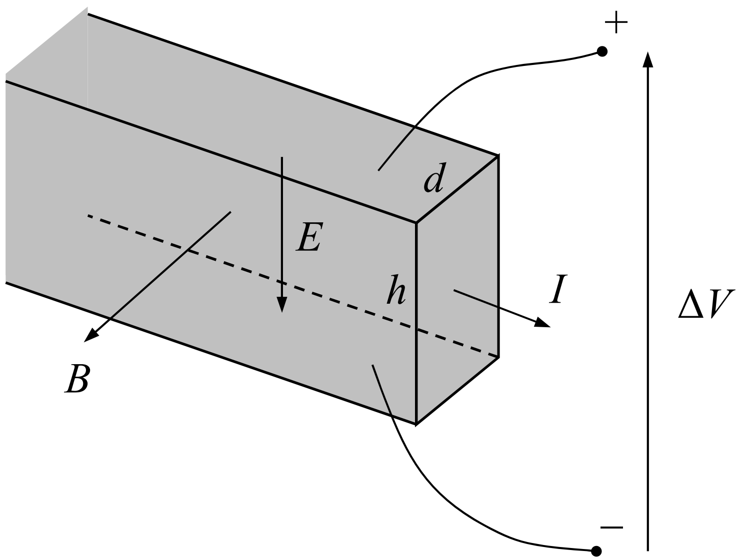
$$W = \int \vec{F}_m \cdot \mathrm{d}\vec{r} = \Delta K = 0$$

$$K = \frac{1}{2} m v^2 = \text{costante}$$

la forza magnetica non compie lavoro

Forza magnetica: effetto Hall

$$F_m = F_e \qquad qvB = qE = q\frac{\Delta V}{h}$$



$$\Delta V = vBh$$

il segno dipende dal segno dei portatori

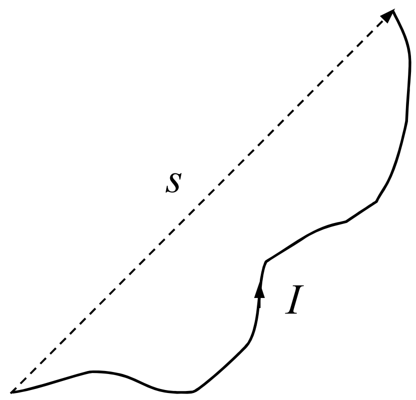
$$J = nqv = \frac{I}{hd} \qquad \Delta V = vBh = \frac{I}{dnq}B$$

$$\frac{\Delta V}{I} = \frac{B}{dnq}$$

resistenza Hall

Forza magnetica: II legge elementare di Laplace

$$\mathrm{d}\vec{F} = \mathrm{d}q\,\vec{v}\times\vec{B} = \mathrm{d}q\frac{\mathrm{d}\vec{s}}{\mathrm{d}t}\times\vec{B} = \frac{\mathrm{d}q}{\mathrm{d}t}\mathrm{d}\vec{s}\times\vec{B} = I\mathrm{d}\vec{s}\times\vec{B}$$



II legge elementare di Laplace

$$\vec{F} = \int \mathrm{d}\vec{F} = I\int \mathrm{d}\vec{s}\times\vec{B}$$

$$\vec{F} = I\int \mathrm{d}\vec{s}\times\vec{B} = I\left(\int \mathrm{d}\vec{s}\right)\times\vec{B} = I\,\vec{s}\times\vec{B}$$

campo uniforme

$$\vec{F} = I\int \mathrm{d}\vec{s}\times\vec{B} = I\left(\oint \mathrm{d}\vec{s}\right)\times\vec{B} = 0$$

circuito chiuso

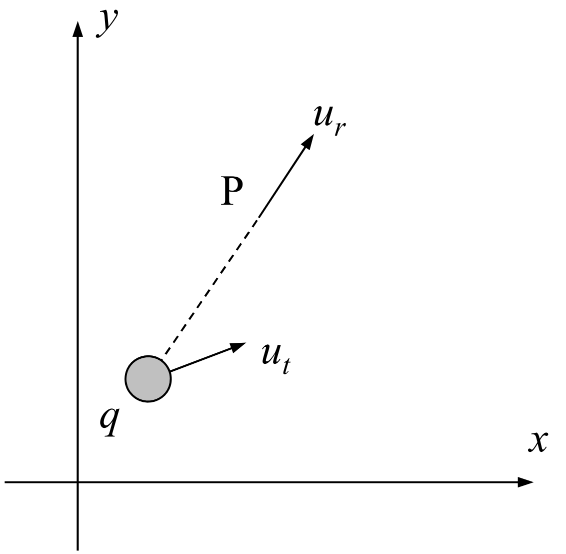
Campo magnetico

$$\vec{B} = \frac{\mu_0}{4\pi} q v \frac{\vec{u}_t \times \vec{u}_r}{r^2}$$

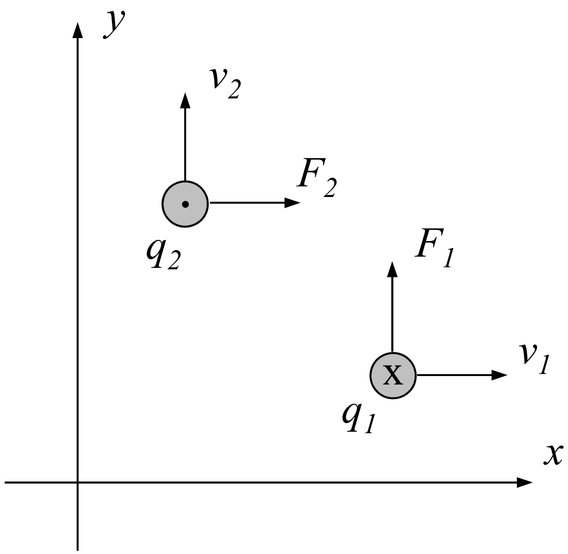
campo magnetico

$$[B] = \mathrm{T}$$

tesla



$$\vec{F}_m = q\vec{v} \times \vec{B}$$

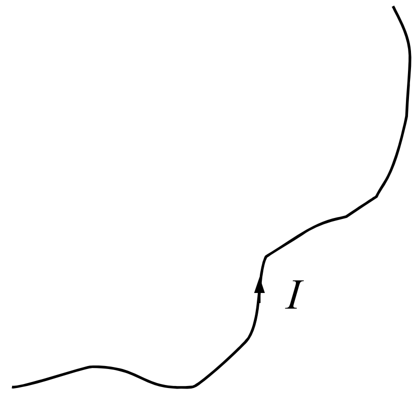


non vale il III principio della dinamica

Campo magnetico: I legge elementare di Laplace

$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}\mathrm{d}q\, v\frac{\vec{u}_t \times \vec{u}_r}{r^2} = \frac{\mu_0}{4\pi}\mathrm{d}q\frac{\mathrm{d}s}{\mathrm{d}t}\frac{\vec{u}_t \times \vec{u}_r}{r^2} =$$

$$= \frac{\mu_0}{4\pi}\frac{\mathrm{d}q}{\mathrm{d}t}\mathrm{d}s\frac{\vec{u}_t \times \vec{u}_r}{r^2} = \frac{\mu_0}{4\pi}I\,\mathrm{d}s\frac{\vec{u}_t \times \vec{u}_r}{r^2}$$

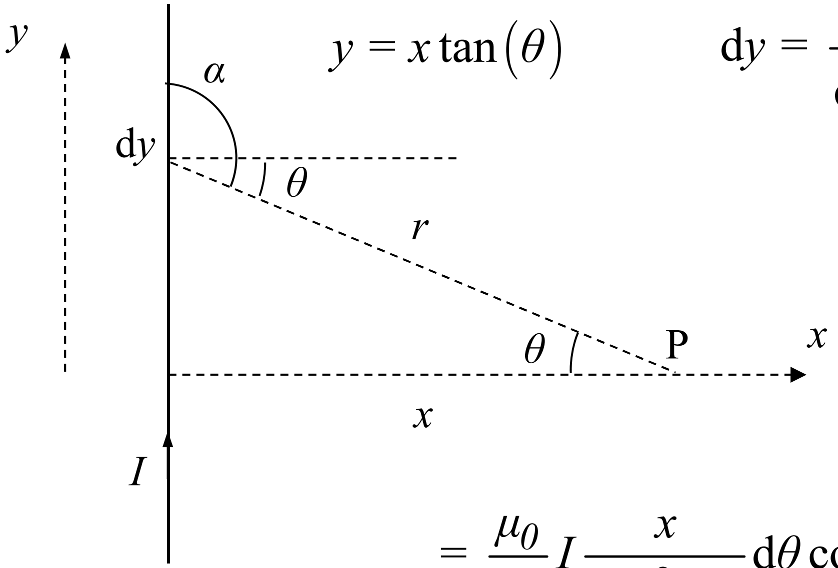


I legge elementare di Laplace

$$\vec{B} = \int \mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}I\int \mathrm{d}s\frac{\vec{u}_t \times \vec{u}_r}{r^2}$$

Campo magnetico: I legge elementare di Laplace

filo rettilineo infinito



$$y = x\tan(\theta) \qquad \mathrm{d}y = \frac{x}{\cos^2(\theta)}\mathrm{d}\theta \qquad \sin(\alpha) = \sin\left(\frac{\pi}{2}+\theta\right) = \cos(\theta)$$

$$\mathrm{d}B = \frac{\mu_0}{4\pi} I\,\mathrm{d}y \frac{\sin(\alpha)}{r^2} =$$

$$r = \frac{x}{\cos(\theta)}$$

$$= \frac{\mu_0}{4\pi} I \frac{x}{\cos^2(\theta)}\mathrm{d}\theta\cos(\theta)\frac{\cos^2(\theta)}{x^2} = \frac{\mu_0}{4\pi}\frac{I}{x}\cos(\theta)\mathrm{d}\theta$$

$$B = \int \mathrm{d}B = \frac{\mu_0}{4\pi}\frac{I}{x}\int_{-\pi/2}^{+\pi/2}\cos(\theta)\mathrm{d}\theta = \frac{\mu_0}{4\pi}\frac{I}{x}\left[\sin(\theta)\right]_{-\pi/2}^{+\pi/2} = \frac{\mu_0}{2\pi}\frac{I}{x}$$

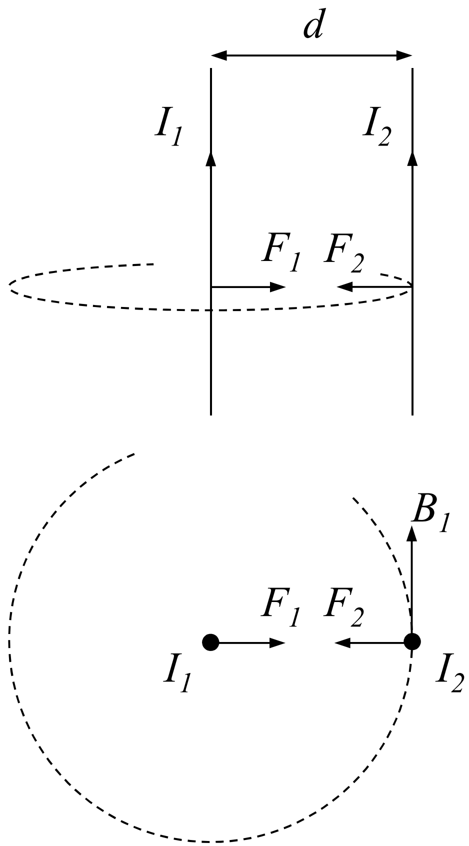
legge di Biot-Savart

Campo magnetico: forza tra correnti

$$B_1 = \frac{\mu_0}{2\pi d} I_1$$

$$F_2 = I_2 L B_1 = I_2 L \frac{\mu_0}{2\pi d} I_1 = L \frac{\mu_0}{2\pi d} I_1 I_2$$

$$\frac{F_2}{L} = \frac{\mu_0}{2\pi d} I_1 I_2$$



vale il III principio della dinamica

Teorema di Gauss

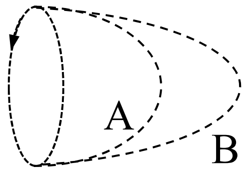
$$\Phi\left(\vec{B}\right) = \oint \vec{B} \cdot \mathrm{d}\vec{S} = 0$$

teorema di Gauss

le linee del campo sono chiuse

non vi sono poli magnetici isolati

$$\Phi\left(\vec{B}\right) = \int_A \vec{B} \cdot \mathrm{d}\vec{S} = \int_B \vec{B} \cdot \mathrm{d}\vec{S}$$



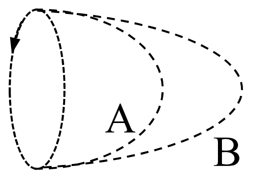
$$\left[\Phi\left(\vec{B}\right)\right] = [B][S] = \mathrm{Tm}^2 = \mathrm{Wb}$$

weber

Teorema di Gauss

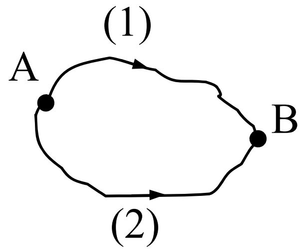
$$\Phi\left(\vec{B}\right) = \oint \vec{B}\cdot \mathrm{d}\vec{S} = 0$$

$$\Phi\left(\vec{B}\right) = \int_{A} \vec{B}\cdot \mathrm{d}\vec{S} = \int_{B} \vec{B}\cdot \mathrm{d}\vec{S}$$



$$\Lambda\left(\vec{E}\right) = \oint \vec{E}\cdot \mathrm{d}\vec{r} = 0$$

$$\Lambda\left(\vec{E}\right) = \underset{(1)}{\int_{A}^{B}} \vec{E}\cdot \mathrm{d}\vec{r} = \underset{(2)}{\int_{A}^{B}} \vec{E}\cdot \mathrm{d}\vec{r}$$

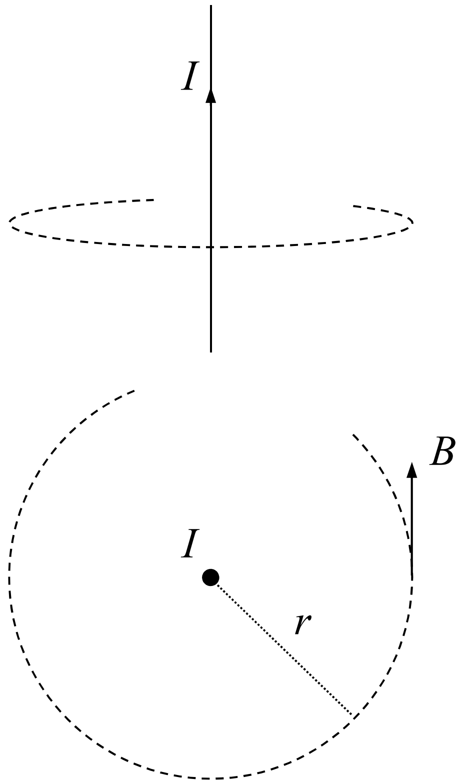


Teorema di Ampère: linea circolare

$$L = 2\pi r$$

$$\Lambda\left(\vec{B}\right) = \oint \vec{B}\cdot \mathrm{d}\vec{r} = \oint B\,\mathrm{d}r = B\oint \mathrm{d}r = BL$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\vec{u}_\theta$$



$$\Lambda\left(\vec{B}\right) = \oint \vec{B}\cdot \mathrm{d}\vec{r} = \mu_0 I$$

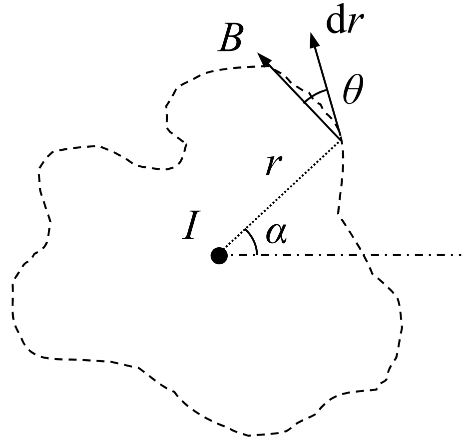
teorema di Ampère

Teorema di Ampère: linea generica

$$\mathrm{d}r\cos(\theta) = \mathrm{d}r' = \mathrm{d}\alpha\, r$$

$$\Lambda\left(\vec{B}\right) = \oint \vec{B}\cdot \mathrm{d}\vec{r} = \oint B\,\mathrm{d}r\cos(\theta)$$

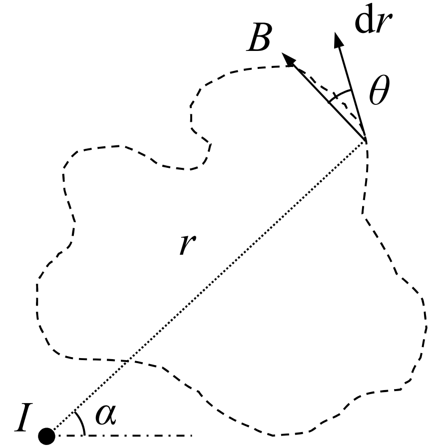
$$\vec{B} = \frac{\mu_0 I}{2\pi r}\vec{u}_\theta$$



$$\Lambda\left(\vec{B}\right) = \oint \mathrm{d}\Lambda = \frac{\mu_0 I}{2\pi}\oint \mathrm{d}\alpha$$

corrente interna $\oint \mathrm{d}\alpha = 2\pi$

corrente esterna $\oint \mathrm{d}\alpha = 0$



Teorema di Ampère

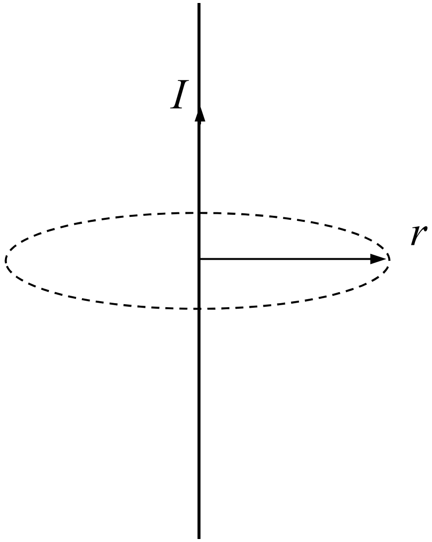
"La circuitazione del campo magnetico creato da più correnti dipende unicamente dalla corrente concatenata dalla linea scelta, e ne risulta proporzionale secondo un fattore μ_0"

$$\Lambda\left(\vec{B}\right) = \oint \vec{B} \cdot \mathrm{d}\vec{r} = \mu_0 I_c$$

sempre valido, non sempre utile

Teorema di Ampère

filo rettilineo infinito



$$\Lambda\left(\vec{B}\right) = \oint \vec{B}\cdot \mathrm{d}\vec{r} = \mu_0 I_c$$

circuitaz. Ampère

$$\Lambda\left(\vec{B}\right) = \oint \vec{B}\cdot \mathrm{d}\vec{r} = \oint B\, \mathrm{d}r = B\oint \mathrm{d}r = B\; 2\pi r$$

$$= \mu_0 I_c = \mu_0 I$$

per simmetria, il campo magnetico è

- ortogonale rispetto al filo
- invariante per traslazione lungo il filo
- invariante per rotazione attorno al filo

⇓

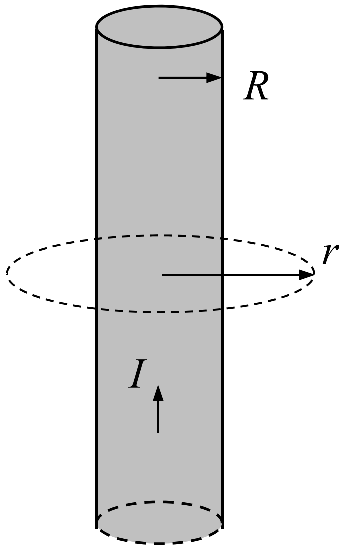
simmetria "cilindrica"

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

legge di Biot-Savart

Teorema di Ampère

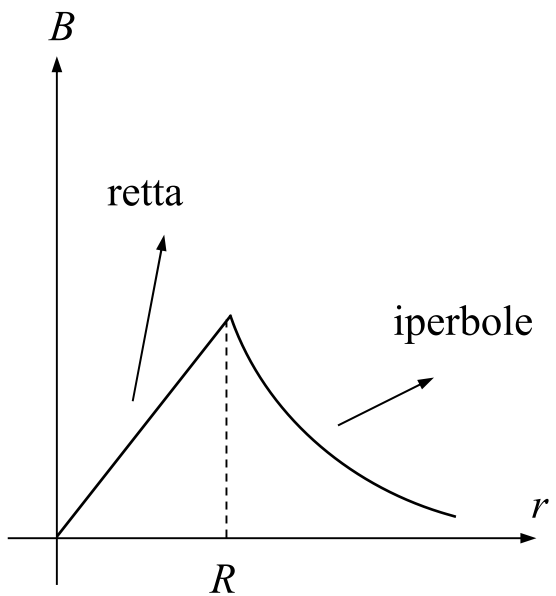
cilindro rettilineo infinito



$$\Lambda\left(\vec{B}\right) = \oint \vec{B}\cdot d\vec{r} = B\,2\pi r = \mu_0 I_c$$

$$r > R: \quad I_c = I \qquad B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

$$r < R: \quad I_c = JS = \frac{I}{\pi R^2}\pi r^2 \qquad B = \frac{\mu_0 I}{2\pi R^2} r$$



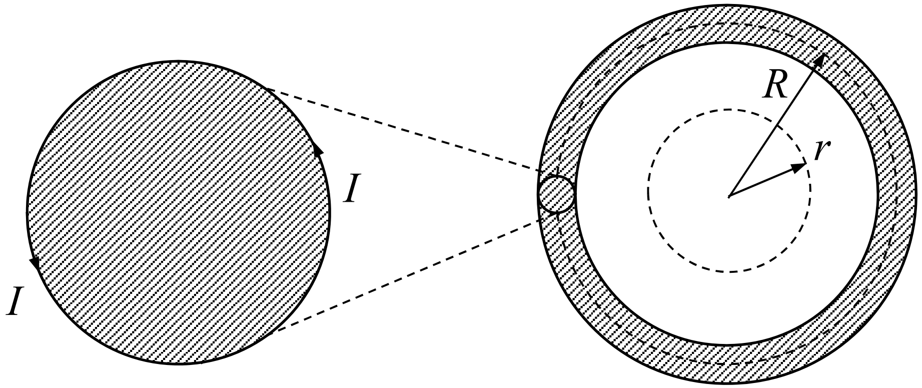
per simmetria, il campo magnetico è

- ortogonale rispetto al filo
- invariante per traslazione lungo il filo
- invariante per rotazione attorno al filo

simmetria "cilindrica"

Teorema di Ampère

solenoide toroidale



$$\Lambda\left(\vec{B}\right) = \oint \vec{B} \cdot \mathrm{d}\vec{r} = B\ 2\pi r = \mu_0 I_c$$

$r \gg R$: $\quad I_c = NI - NI = 0 \qquad B = 0$

$r \ll R$: $\quad I_c = 0 \qquad B = 0$

$r \approx R$: $\quad I_c = NI \qquad B = \dfrac{\mu_0 NI}{2\pi r} = \mu_0 n I$

per simmetria, il campo magnetico è

- invariante per rotazione

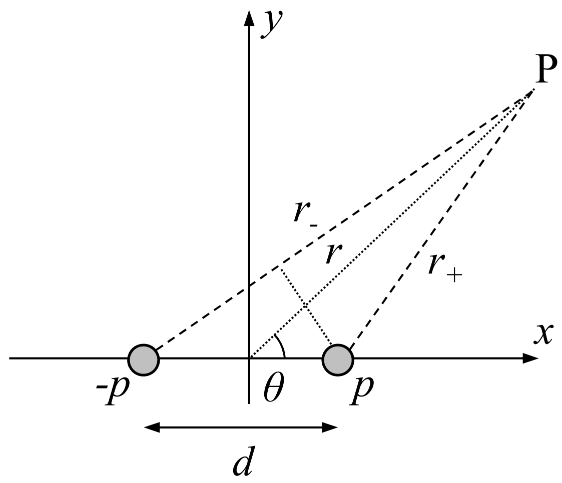
simmetria rotazionale

$$n = \frac{N}{2\pi r}$$

densità di spire

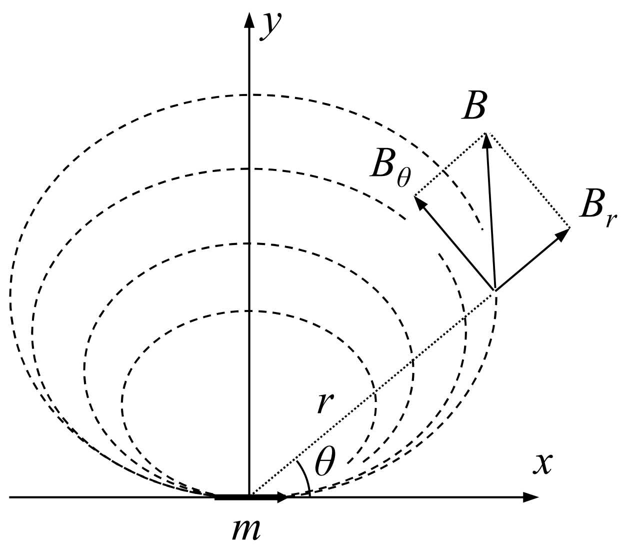
Dipolo magnetico: interazioni create

$$\begin{cases} B_r = \dfrac{\mu_0}{4\pi}\dfrac{2m\cos(\theta)}{r^3} \\ B_\theta = \dfrac{\mu_0}{4\pi}\dfrac{m\sin(\theta)}{r^3} \end{cases}$$



$$\vec{m} = p\vec{d} \qquad [m] = [p][d] = \text{Am}^2$$

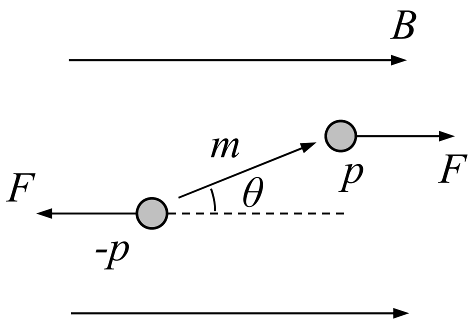
momento di dipolo magnetico



Dipolo magnetico: interazioni subite

$$\vec{M} = \vec{d} \times \vec{F} = \vec{d} \times p\vec{B} = p\vec{d} \times \vec{B} = \vec{m} \times \vec{B}$$

$$U = -\vec{m} \cdot \vec{B}$$



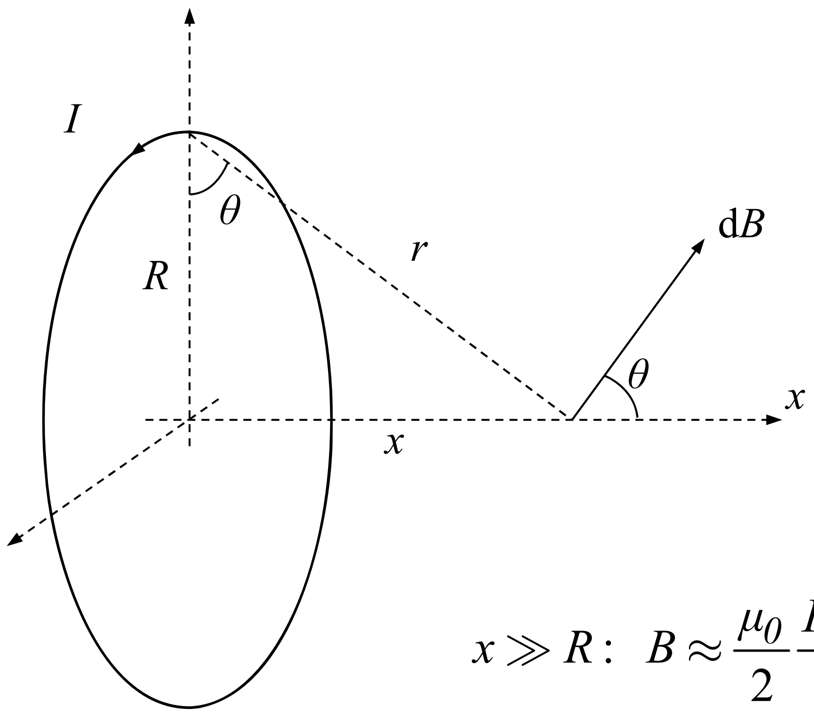
$$\vec{F} = -\operatorname{grad}\left(U\right) = \operatorname{grad}\left(\vec{m} \cdot \vec{B}\right)$$

Dipolo magnetico: spira circolare

$$d\vec{B} = \frac{\mu_0}{4\pi} I\, ds \frac{\vec{u}_t \times \vec{u}_r}{r^2}$$

$$dB_x = dB\cos\theta = \frac{\mu_0}{4\pi}\frac{I\, ds}{r^2}\cos(\theta) =$$

$$= \frac{\mu_0}{4\pi} I\, ds \frac{1}{x^2 + R^2}\frac{R}{\sqrt{x^2 + R^2}}$$



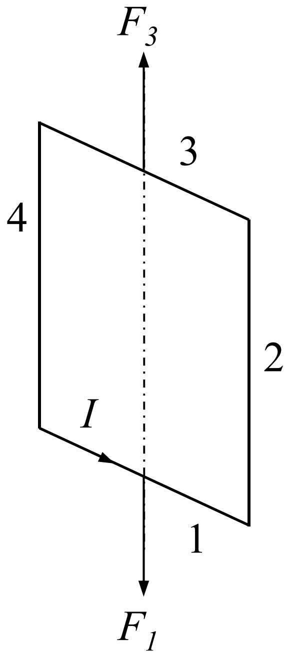
$$B = \int dB_x = \frac{\mu_0}{2}\frac{IR^2}{\left(x^2 + R^2\right)^{3/2}}$$

$$x \gg R: \quad B \approx \frac{\mu_0}{2}\frac{IR^2}{x^3} = \frac{\mu_0}{4\pi}\frac{2\cdot I\pi R^2}{x^3} = \frac{\mu_0}{4\pi}\frac{2m}{x^3}$$

$$m = IS = I\pi R^2 \quad \Rightarrow \quad \vec{m} = I\vec{S}$$

momento di dipolo magnetico

Dipolo magnetico: spira rettangolare



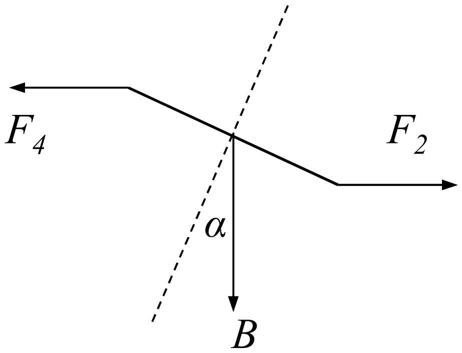
$$\vec{F} = I\int d\vec{s} \times \vec{B} = I\,\vec{s} \times \vec{B}$$

$$\begin{cases} F_1 = F_3 = I d_1 B \sin\left(\dfrac{\pi}{2} - \alpha\right) \\ F_2 = F_4 = I d_2 B \end{cases}$$

$\Rightarrow$ $F_{tot} = 0$

$$S = d_1 d_2$$

$$M = d_1 F_2 \sin(\alpha) = d_1 I d_2 B \sin(\alpha) = IS \cdot B \sin(\alpha) = mB\sin(\alpha)$$

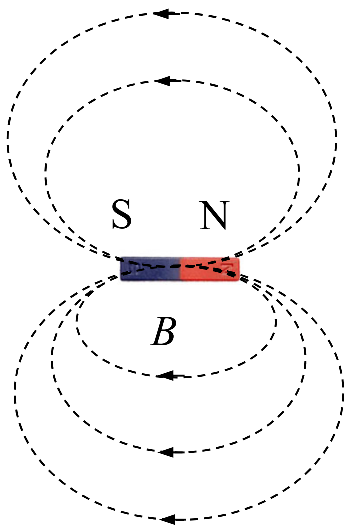


$$\vec{M} = \vec{m} \times \vec{B}$$

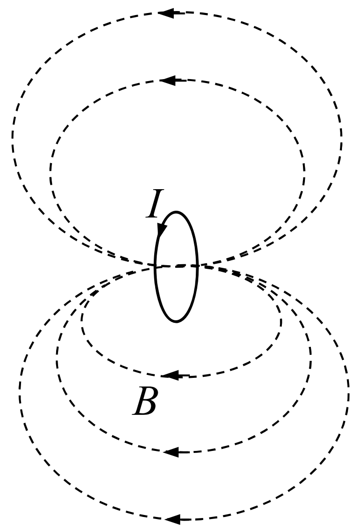
Teorema di equivalenza di Ampère

"Il campo magnetico creato e le interazioni subite
da un magnete e da una spira (in approx. di dipolo)
sono equivalenti"

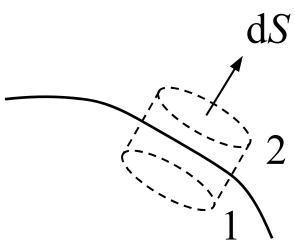
magnete



corrente



Formulazione differenziale: condizioni al contorno



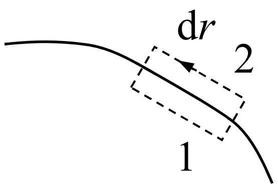
$$
\begin{cases}
d\Phi_2(\vec{B}) = \vec{B}_2 \cdot d\vec{S}_2 = B_{n2} dS \\
d\Phi_1(\vec{B}) = \vec{B}_1 \cdot d\vec{S}_1 = -\vec{B}_1 \cdot d\vec{S}_2 = -B_{n1} dS \\
d\Phi_{lat}(\vec{B}) \approx 0
\end{cases}
$$

$$
d\Phi(\vec{B}) = d\Phi_1(\vec{B}) + d\Phi_2(\vec{B}) + d\Phi_{lat}(\vec{B}) = (B_{n2} - B_{n1}) dS = \Delta B_n dS
$$

$$
d\Phi(\vec{B}) = 0
$$

$$
\Delta B_n = 0
$$

Formulazione differenziale: condizioni al contorno



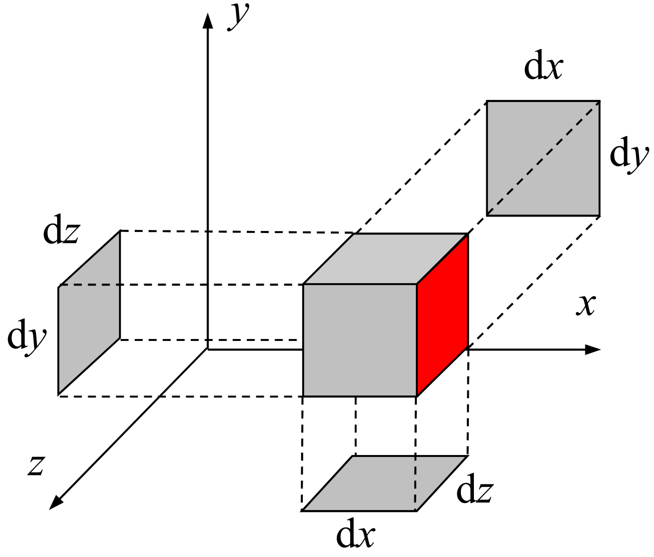
$$
\begin{cases}
d\Lambda_2\left(\vec{B}\right) = \vec{B}_2 \cdot d\vec{r}_2 = B_{t2}dr \\
d\Lambda_1\left(\vec{B}\right) = \vec{B}_1 \cdot d\vec{r}_1 = -\vec{B}_1 \cdot d\vec{r}_2 = -B_{t1}dr \\
d\Lambda_n\left(\vec{B}\right) \approx 0
\end{cases}
$$

$$d\Lambda\left(\vec{B}\right) = d\Lambda_1\left(\vec{B}\right) + d\Lambda_2\left(\vec{B}\right) + d\Lambda_n\left(\vec{B}\right) = \left(B_{t2} - B_{t1}\right)dr = \Delta B_t dr$$

$$d\Lambda\left(\vec{B}\right) = \mu_0 dI = \mu_0 K_b dr$$

$$\Delta B_t = \mu_0 K_b$$

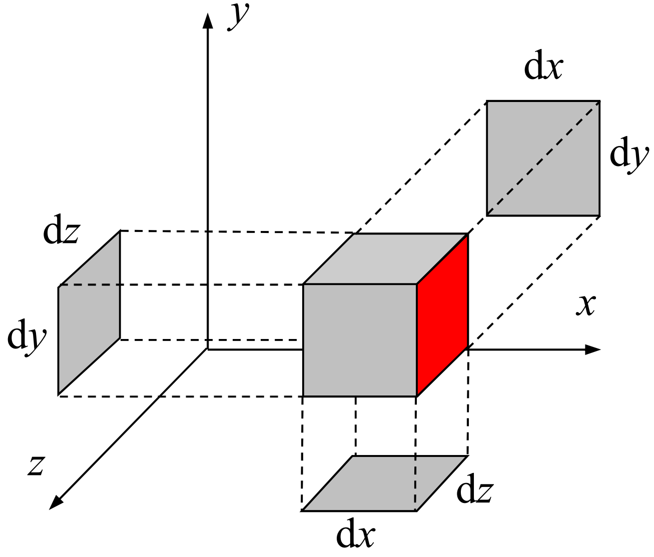
Formulazione differenziale: leggi di Maxwell



$$\begin{cases} d\Phi_x''\left(\vec{B}\right) = \vec{B}'' \cdot d\vec{S} = B_x'' dS = B_x'' dy dz \\ d\Phi_x'\left(\vec{B}\right) = \vec{B}_x' \cdot d\vec{S} = -B_x' dS = -B_x' dy dz \end{cases}$$

$$\begin{cases} d\Phi_x\left(\vec{B}\right) = d\Phi_x''\left(\vec{B}\right) + d\Phi_x'\left(\vec{B}\right) = B_x'' dy dz - B_x' dy dz = dB_x dy dz = \left(\frac{\partial B_x}{\partial x} dx\right) dy dz = \frac{\partial B_x}{\partial x} dV \\ d\Phi_y\left(\vec{B}\right) = \frac{\partial B_y}{\partial y} dV \\ d\Phi_z\left(\vec{B}\right) = \frac{\partial B_z}{\partial z} dV \end{cases}$$

Formulazione differenziale: leggi di Maxwell

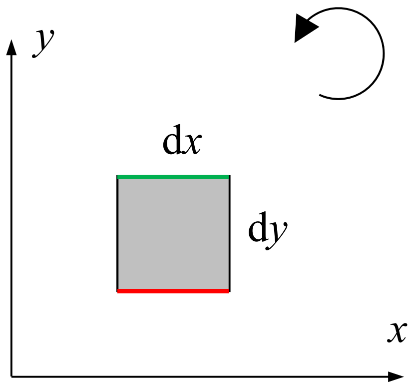


$$d\Phi\left(\vec{B}\right) = d\Phi_x\left(\vec{B}\right) + d\Phi_y\left(\vec{B}\right) + d\Phi_z\left(\vec{B}\right) = \left(\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}\right) dV = \operatorname{div}\left(\vec{B}\right) dV$$

$$d\Phi\left(\vec{B}\right) = 0$$

$$\operatorname{div}\left(\vec{B}\right) = 0$$

Formulazione differenziale: leggi di Maxwell

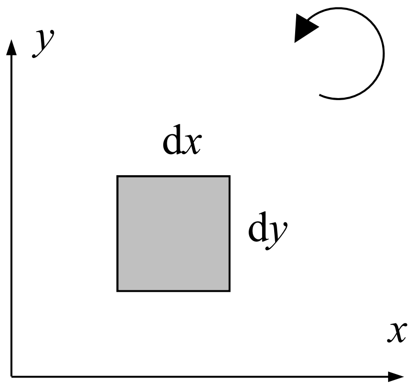


$$\begin{cases} d\Lambda_x'\left(\vec{B}\right) = \vec{B}'\cdot d\vec{r} = B_x' dx \\ d\Lambda_x''\left(\vec{B}\right) = \vec{B}''\cdot d\vec{r} = -B_x'' dx \end{cases}$$

$$\begin{cases} d\Lambda_x\left(\vec{B}\right) = d\Lambda_x'\left(\vec{B}\right) + d\Lambda_x''\left(\vec{B}\right) = B_x' dx - B_x'' dx = -dB_x dx = -\left(\frac{\partial B_x}{\partial y} dy\right) dx \\ d\Lambda_y\left(\vec{B}\right) = dB_y dy = \frac{\partial B_y}{\partial x} dx dy \end{cases}$$

$$d\Lambda\left(\vec{B}\right) = d\Lambda_y\left(\vec{B}\right) + d\Lambda_x\left(\vec{B}\right) = \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) dx dy$$

Formulazione differenziale: leggi di Maxwell

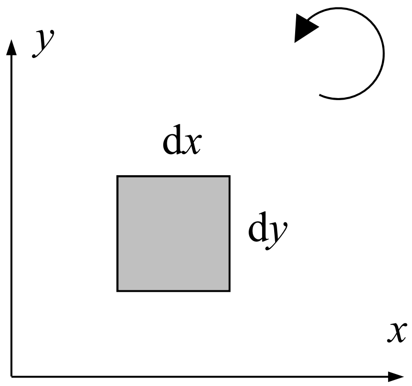


piano *xy*: $$\mathrm{d}\Lambda\left(\vec{B}\right) = \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)\mathrm{d}x\mathrm{d}y = \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)\mathrm{d}S_z$$

piano *yz*: $$\mathrm{d}\Lambda\left(\vec{B}\right) = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right)\mathrm{d}y\mathrm{d}z = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right)\mathrm{d}S_x$$

piano *zx*: $$\mathrm{d}\Lambda\left(\vec{B}\right) = \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right)\mathrm{d}z\mathrm{d}x = \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right)\mathrm{d}S_y$$

Formulazione differenziale: leggi di Maxwell



$$\mathrm{d}\Lambda\left(\vec{B}\right) = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right)\mathrm{d}S_x + \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right)\mathrm{d}S_y + \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)\mathrm{d}S_z = \mathrm{rot}\left(\vec{B}\right)\cdot \mathrm{d}\vec{S}$$

$$\mathrm{d}\Lambda\left(\vec{B}\right) = \mu_0 \mathrm{d}I = \mu_0 \vec{J}\cdot \mathrm{d}\vec{S}$$

$$\mathrm{rot}\left(\vec{B}\right) = \mu_0 \vec{J}$$

Formulazione differenziale: leggi di Maxwell

	Teorema di Gauss	Teorema di Ampère
relazioni integrali	$\Phi\left(\vec{B}\right) = \oint \vec{B}\cdot \mathrm{d}\vec{S} = 0$	$\Lambda\left(\vec{B}\right) = \oint \vec{B}\cdot \mathrm{d}\vec{r} = \mu_0 I_c$
condizioni al contorno	$\Delta B_n = 0$	$\Delta B_t = \mu_0 K_b$
relazioni infinitesime	$\mathrm{div}\left(\vec{B}\right) = 0$	$\mathrm{rot}\left(\vec{B}\right) = \mu_0 \vec{J}$

Formulazione differenziale: leggi di Maxwell

$$\begin{cases} \mathrm{div}\left(\vec{B}\right) = 0 \\ \mathrm{rot}\left(\vec{B}\right) = \mu_0 \vec{J} \end{cases}$$

$$\mathrm{div}\left(\mathrm{rot}\left(\vec{v}\right)\right) \equiv 0 \quad \Rightarrow \quad \vec{B} = \mathrm{rot}\left(\vec{A}\right)$$

$$\vec{A} = ?$$

$$\vec{B} = \int \mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}\int \mathrm{rot}\left(\frac{\vec{J}}{r}\right)\mathrm{d}V = \mathrm{rot}\left(\frac{\mu_0}{4\pi}\int \frac{\vec{J}}{r}\mathrm{d}V\right)$$

$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi} I\,\mathrm{d}s\,\frac{\vec{u}_t \times \vec{u}_r}{r^2} = \frac{\mu_0}{4\pi} JS\,\mathrm{d}s\,\frac{\vec{u}_t \times \vec{u}_r}{r^2} =$$

$$= \frac{\mu_0}{4\pi} J\mathrm{d}V\,\frac{\vec{u}_t \times \vec{u}_r}{r^2} = \frac{\mu_0}{4\pi}\mathrm{d}V\,\frac{\vec{J} \times \vec{u}_r}{r^2} =$$

$$= \frac{\mu_0}{4\pi}\mathrm{d}V\,\mathrm{rot}\left(\frac{\vec{J}}{r}\right)$$

potenziale vettore

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}}{r}\mathrm{d}V$$

vale la sovrapposizione degli effetti

Formulazione differenziale: leggi di Maxwell

$$\begin{cases} \mathrm{div}\left(\vec{B}\right) = 0 \\ \mathrm{rot}\left(\vec{B}\right) = \mu_0 \vec{J} \end{cases}$$

$$\mathrm{div}\left(\mathrm{rot}\left(\vec{v}\right)\right) \equiv 0 \quad \Rightarrow \quad \vec{B} = \mathrm{rot}\left(\vec{A}\right)$$

arbitraria

$$\vec{A} = \vec{A}_0 + \mathrm{grad}\left(f\right)$$

$$\mathrm{rot}\left(\vec{A}\right) = \mathrm{rot}\left(\vec{A}_0 + \mathrm{grad}\left(f\right)\right) =$$

$$= \mathrm{rot}\left(\vec{A}_0\right) + \mathrm{rot}\left(\cancel{\mathrm{grad}}\left(f\right)\right) = \mathrm{rot}\left(\vec{A}_0\right)$$

$$\mathrm{rot}\left(\vec{B}\right) = \mathrm{rot}\left(\mathrm{rot}\left(\vec{A}\right)\right) =$$

$$= \mathrm{grad}\left(\cancel{\mathrm{div}\left(\vec{A}\right)}\right) - \nabla^2\left(\vec{A}\right) = -\nabla^2\left(\vec{A}\right) = \mu_0 \vec{J}$$

gauge di Coulomb

equazione di Poisson