



POLITECNICO
MILANO 1863

Elettromagnetismo

Elettricità. Corrente. Magnetismo

Maurizio Zani

Elettromagnetismo

[Elettrostatica](#)

[Materiali conduttori](#)

[Condensatori](#)

[Materiali dielettrici](#)

[Corrente elettrica](#)

[Resistori](#)

[Circuiti elettrici continui](#)

[Magnetostatica](#)

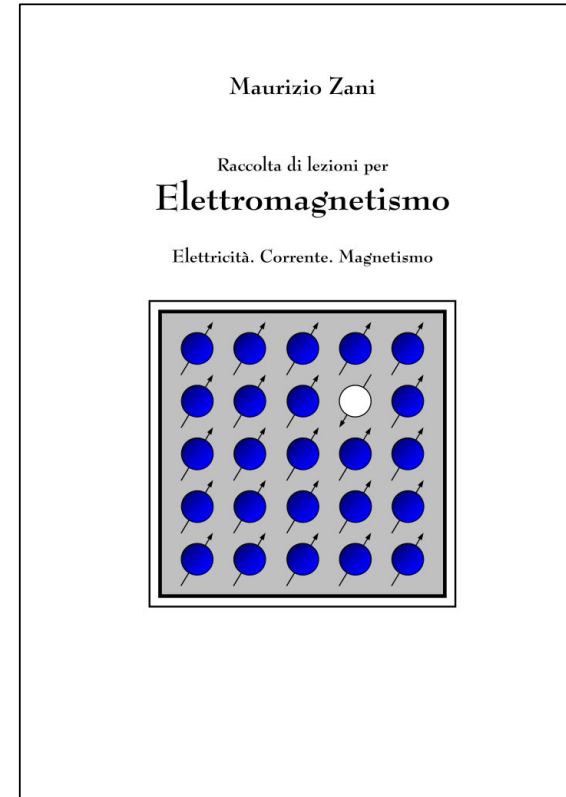
[Induzione elettromagnetica](#)

[Induttori](#)

[Materiali magnetici](#)

[Circuiti elettrici variabili](#)

[Elettromagnetismo](#)



<http://www.mauriziozani.it/wp/?p=1128>



Elettromagnetismo

Elettrostatica

Materiali conduttori

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Elettromagnetismo

Elettrizzazione

Forza elettrica

Campo elettrico

Teorema di Gauss

Campo conservativo

Dipolo elettrico

Formulazione differenziale



Elettrizzazione: strofinio



il materiale si elettrizza?

- no: **materiale conduttore**
 - sì: **materiale isolante**
 - come la plastica (-): **elettrizzazione resinosa**
 - come il vetro (+): **elettrizzazione vetrosa**
- carica
- tipo diverso: attrazione
stesso tipo: repulsione



Elettrizzazione: strofinio



vetro (+)

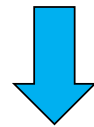
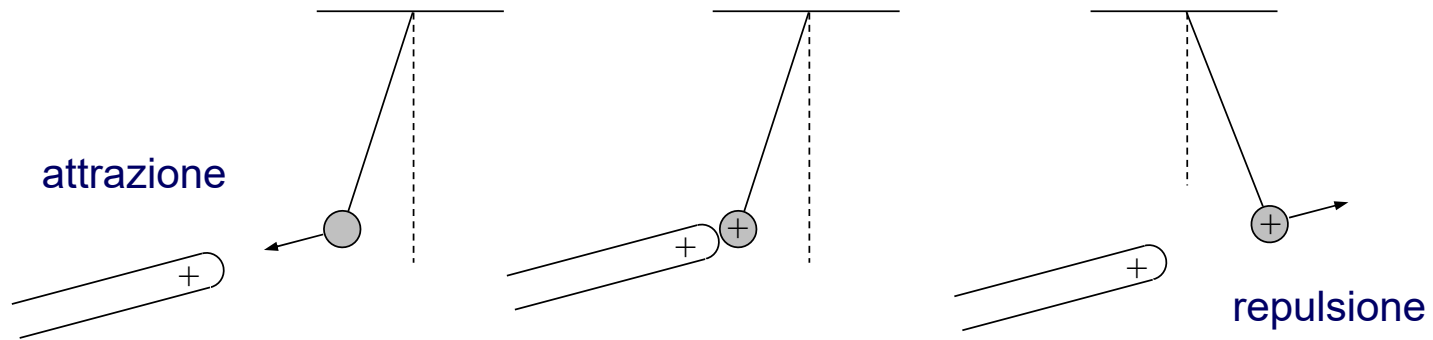


plastica (-)

Si	H	C
elettronegatività		
1.90	2.20	2.55



materiali conduttori

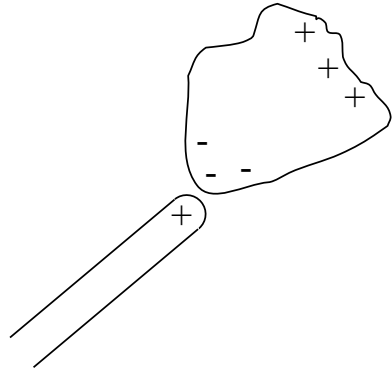


contatto

l'interazione
cambia segno

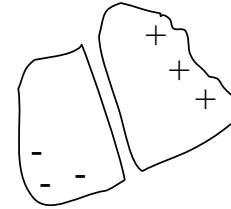
Elettrizzazione: induzione elettrostatica

materiali conduttori



prossimità (senza contatto)

elettrizzazione
localizzata e temporanea



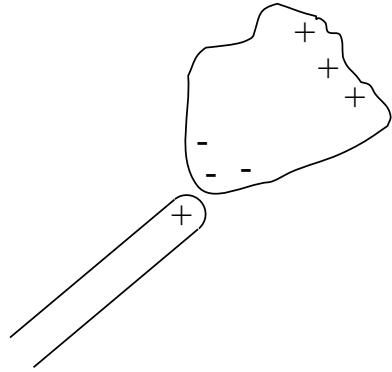
prossimità e taglio

elettrizzazione
opposta e permanente



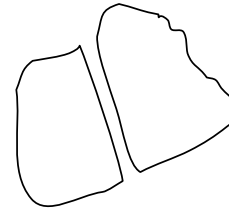
Elettrizzazione: polarizzazione

materiali isolanti



prossimità (senza contatto)

elettrizzazione
localizzata e temporanea



prossimità e taglio

nessuna
elettrizzazione

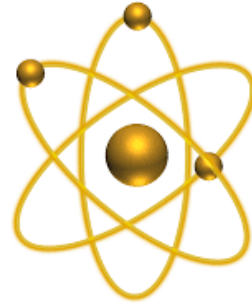


q

carica elettrica

$$[q] = [I][t] = \text{As} = \text{C}$$

coulomb



protone

- $m_p = 1.672622 \cdot 10^{-27} \text{ kg}; \quad q_p = 1.602176 \cdot 10^{-19} \text{ C}$

elettrone

- $m_e = 9.109382 \cdot 10^{-31} \text{ kg}; \quad q_e = -1.602176 \cdot 10^{-19} \text{ C}$

neutrone

- $m_n = 1.674927 \cdot 10^{-27} \text{ kg}; \quad q_n = 0 \text{ C}$

Modello di Thomson (1902)

- atomo come sfera carica positivamente (e senza massa)
- elettroni cariche negative al suo interno (con massa)

Modello di Lorentz (1905)

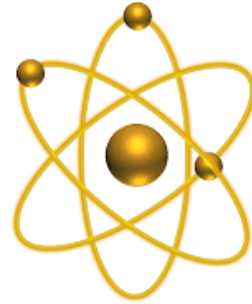
- nucleo carico positivamente
- elettroni come sfera carica negativamente

Modello di Rutherford (1911)

- nucleo carico positivamente
- elettroni cariche negative che orbitano

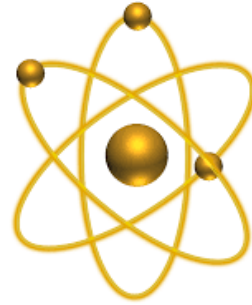
Modello di Bohr (1913)

- nucleo carico positivamente
- elettroni cariche negative su orbite stazionarie



Atomo

- atomo: $r = 10^{-10}$ m
- nucleo: $r = 10^{-15}$ m
- elettrone: $r = 10^{-18}$ m



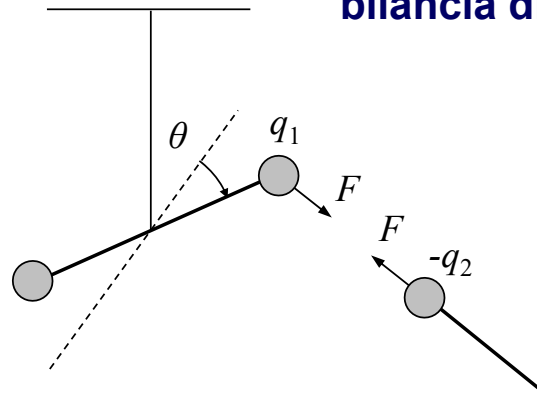
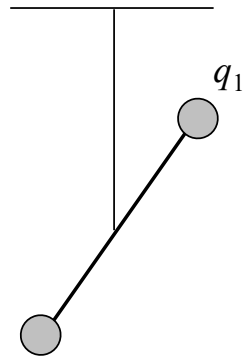
$\cdot 10^{17}$

- atomo $\Rightarrow 10^7$ m (pianeta Terra)
- nucleo $\Rightarrow 10^2$ m (campo da calcio)
- elettrone $\Rightarrow 10^{-1}$ m (pallone da calcio)



l'atomo è vuoto!

Forza elettrica: forza di Coulomb



bilancia di torsione

$$\begin{cases} M_t = k_t \theta \\ M_e = F_e b \end{cases}$$

$$F_e = \frac{k_t \theta}{b}$$

costante elettrica

$$\vec{F}_e = k_e \frac{q_1 q_2}{r^2} \vec{u}_r$$

forza elettrica (di Coulomb)

forza fondamentale

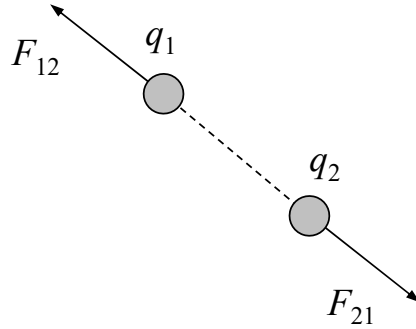
$$k_e = \frac{1}{4\pi\epsilon_0} = 8.9874 \cdot 10^9 \text{ Nm}^2 / \text{C}^2$$

$$\epsilon_0 = 8.85418781762 \cdot 10^{-12} \text{ C}^2 / \text{Nm}^2$$

permittività elettrica

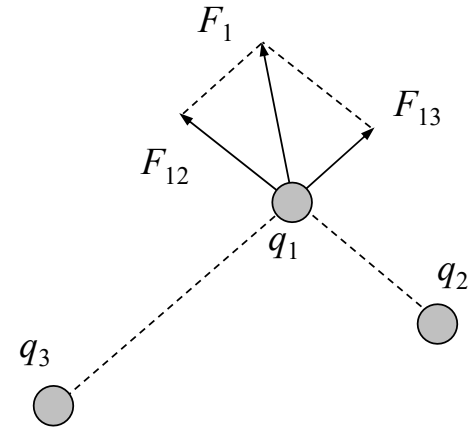


Forza elettrica: forza di Coulomb



azione e reazione

$$|\vec{F}_{12}| = |\vec{F}_{21}|$$



sovrapposizione degli effetti

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13}$$



Forza elettrica: forza di Coulomb

$$\vec{F}_g = -\gamma \frac{m_p m_e}{r^2} \vec{u}_r$$

$$\frac{F_e}{F_g} = \frac{k_e}{\gamma} \frac{q_p q_e}{m_p m_e} \approx 10^{39}$$

confronto con la
forza gravitazionale



$$\vec{F}_e = k_e \frac{q_1 q_2}{r^2} \vec{u}_r$$

forza elettrica (di Coulomb)

- età dell'universo
 - 13 M anni = $4 \cdot 10^{17}$ s
- dimensione dell'universo
 - 92 M anni luce = $8.7 \cdot 10^{26}$ m

interazione tra due cariche
 $q = 1$ C; $r = 1$ m

$$F_e = 9 \cdot 10^9 \text{ N}$$



450 Shuttle!



$$\vec{F}_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \vec{u}_r$$

$$[E] = \frac{[F]}{[q]} = \frac{\text{N}}{\text{C}}$$

$$\vec{F}_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \vec{u}_r = q_1 \left(\frac{1}{4\pi\epsilon_0} \frac{q_2}{r^2} \vec{u}_r \right) = q_1 \vec{E}_2$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \vec{u}_r$$

effetto

oggetto

causa

campo elettrico

$$\vec{F}_l = \sum \vec{F}_i = \sum q_l \vec{E}_i = q_l \sum \vec{E}_i = q_l \vec{E}$$

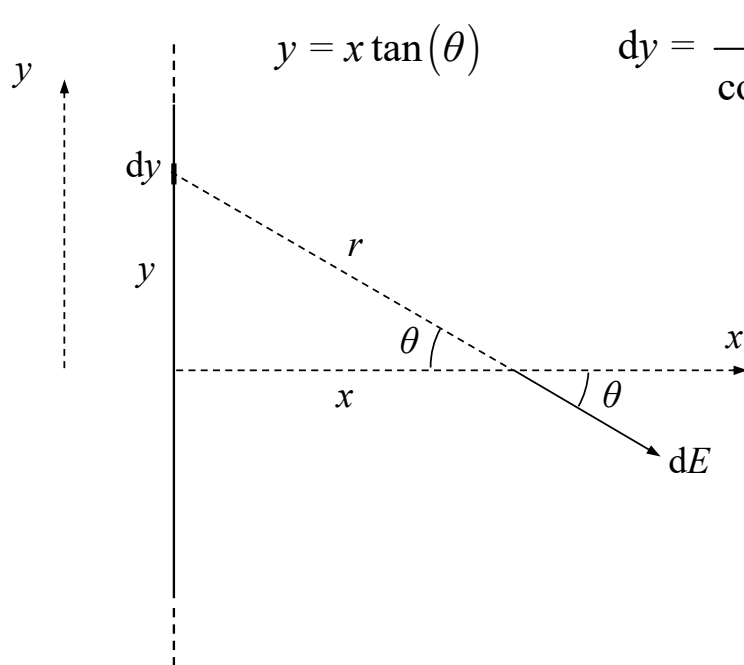
vale la sovrapposizione degli effetti

$$\vec{E} = \sum \vec{E}_i$$

$$\vec{E} = \int d\vec{E}$$



filo rettilineo infinito unif. carico



$$dy = \frac{x}{\cos^2(\theta)} d\theta$$

$$dq = \lambda dy = \lambda \frac{x}{\cos^2(\theta)} d\theta$$

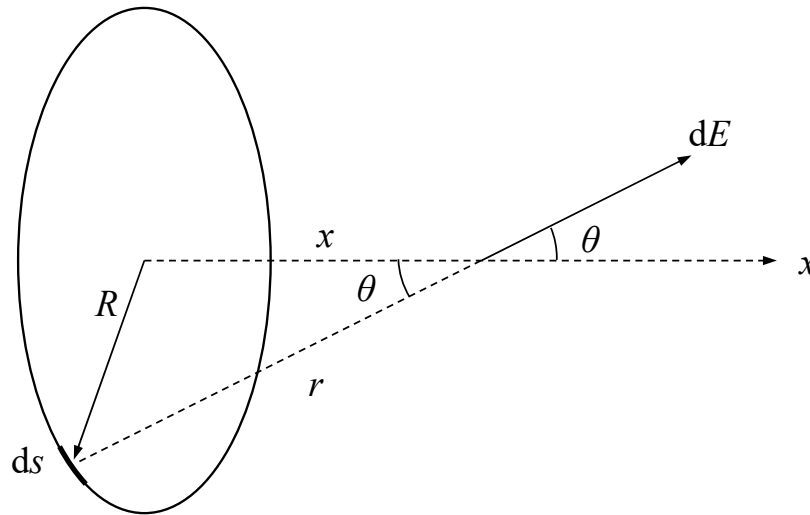
$$dE_x = dE \cos(\theta) = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \cos(\theta) =$$

$$= \frac{1}{4\pi\epsilon_0} \frac{\lambda}{x} \cos(\theta) d\theta$$

$$r = \frac{x}{\cos(\theta)}$$

$$E = \int dE_x = \frac{1}{4\pi\epsilon_0} \frac{\lambda}{x} \int_{-\pi/2}^{+\pi/2} \cos(\theta) d\theta = \frac{1}{4\pi\epsilon_0} \frac{\lambda}{x} [\sin(\theta)]_{-\pi/2}^{+\pi/2} = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{x}$$

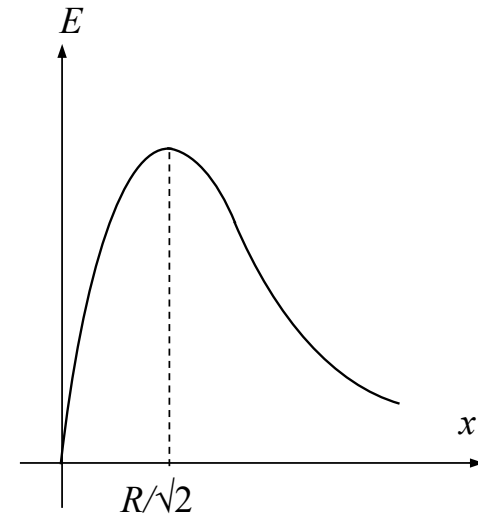
anello unif. carico



$$dE_x = dE \cos(\theta) = \frac{1}{4\pi\epsilon_0} \frac{dq}{x^2 + R^2} \cos(\theta) =$$

$$= \frac{1}{4\pi\epsilon_0} \frac{dq}{x^2 + R^2} \frac{x}{\sqrt{x^2 + R^2}}$$

$$E = \int dE_x = \frac{1}{4\pi\epsilon_0} \frac{xq}{(x^2 + R^2)^{3/2}}$$



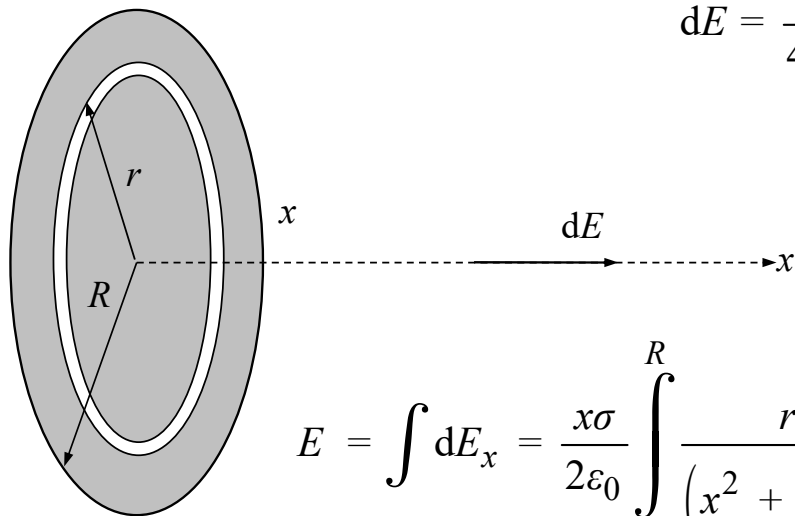
disco unif. carico

$$\sigma = \frac{q_{disco}}{\pi R^2}$$

$$q_{disco} = \sigma \pi r^2$$

$$dq = \sigma 2\pi r dr$$

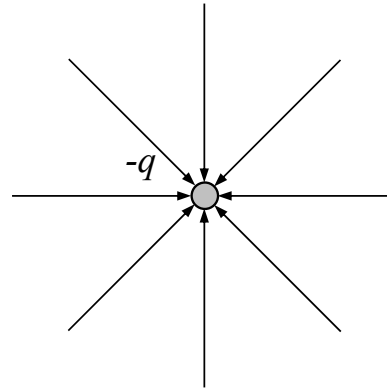
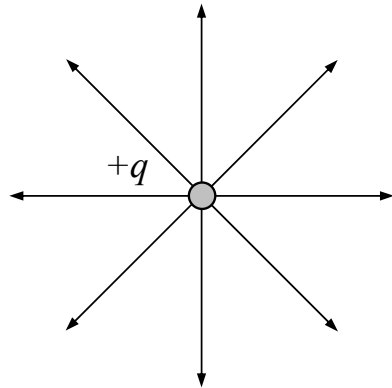
$$dE = \frac{1}{4\pi\epsilon_0} \frac{x dq}{(x^2 + r^2)^{3/2}}$$



$$E = \int dE_x = \frac{x\sigma}{2\epsilon_0} \int_0^R \frac{r}{(x^2 + r^2)^{3/2}} dr = \begin{cases} x \ll R: E \approx \frac{\sigma}{2\epsilon_0} \\ x \gg R: E \approx \frac{1}{4\pi\epsilon_0} \frac{q}{x^2} \end{cases}$$
$$= \frac{\sigma}{2\epsilon_0} \left(1 - \frac{x}{\sqrt{x^2 + R^2}} \right)$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \vec{u}_r$$

$$\vec{E} = \int d\vec{E}$$

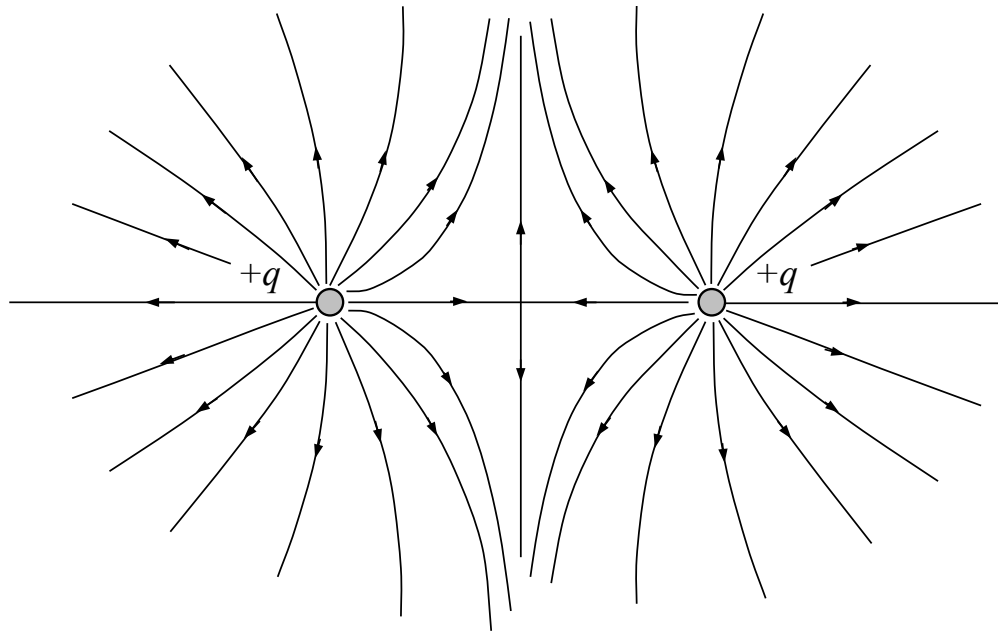


linee di flusso

- linee orientate, tangenti (direzione) e concordi (verso) al campo
- si addensano dove il campo è più intenso
- non si incrociano mai
- partono (sorgente) e terminano (pozzo) sulle cariche o all'infinito

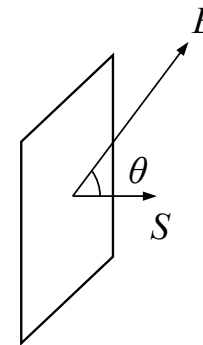
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \vec{u}_r$$

$$\vec{E} = \int d\vec{E}$$



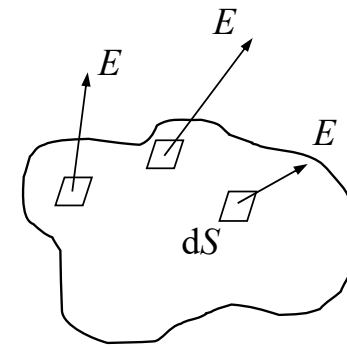
Teorema di Gauss: flusso

campo omogeneo: $\Phi(\vec{E}) = \vec{E} \cdot \vec{S} \quad \left[\Phi(\vec{E}) \right] = [E][S] = \frac{N}{C} \text{m}^2$



flusso

campo/superficie variabile: $\Phi(\vec{E}) = \int d\Phi(\vec{E}) = \int \vec{E} \cdot d\vec{S}$



- flusso additivo tra campi $\Phi(\vec{E}) = \int (\vec{E}_1 + \vec{E}_2) \cdot d\vec{S} = \int \vec{E}_1 \cdot d\vec{S} + \int \vec{E}_2 \cdot d\vec{S} = \Phi_1(\vec{E}) + \Phi_2(\vec{E})$

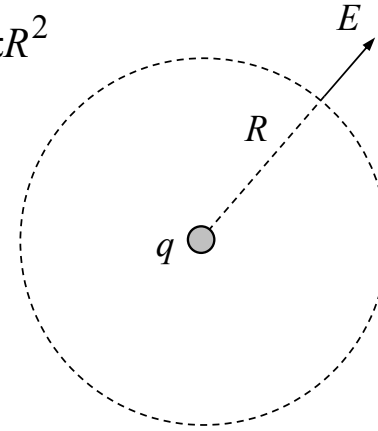
- flusso additivo in superfici $\Phi(\vec{E}) = \int_{S_3} \vec{E} \cdot d\vec{S} = \int_{S_1} \vec{E} \cdot d\vec{S} + \int_{S_2} \vec{E} \cdot d\vec{S} = \Phi_1(\vec{E}) + \Phi_2(\vec{E})$



Teorema di Gauss: superficie sferica

$$\Phi(\vec{E}) = \oint \vec{E} \cdot d\vec{S} = \oint E dS = E \oint dS = E S$$

$$S = 4\pi R^2$$



$$\Phi(\vec{E}) = \oint \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \vec{u}_r$$

teorema di Gauss

$$\Phi(\vec{E}) = \oint \vec{E} \cdot d\vec{S} = 4\pi k_e \cdot q$$

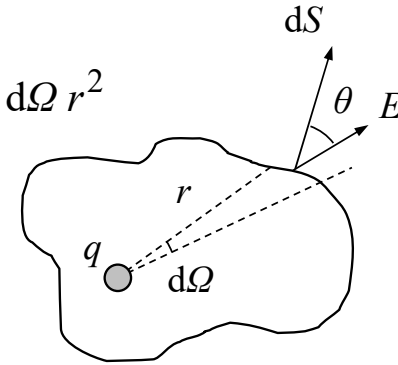
$$\Phi(\vec{G}) = \oint \vec{G} \cdot d\vec{S} = -4\pi\gamma \cdot m$$



Teorema di Gauss: superficie generica

$$\Phi(\vec{E}) = \oint \vec{E} \cdot d\vec{S} = \oint E dS \cos(\theta)$$

$$dS \cos(\theta) = dS' = d\Omega r^2$$

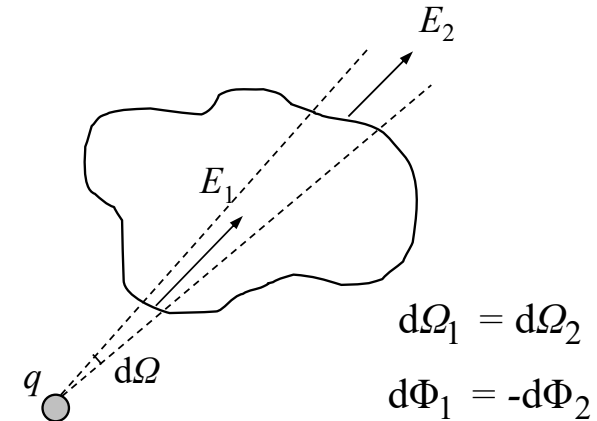


$$\Phi(\vec{E}) = \oint d\Phi = \frac{q}{4\pi\epsilon_0} \oint d\Omega$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \vec{u}_r$$

carica interna $\oint d\Omega = 4\pi$

carica esterna $\oint d\Phi = 0$



Teorema di Gauss

"Il flusso del campo elettrico attraverso una superficie chiusa dipende unicamente dalla carica netta contenuta nella superficie, e ne risulta proporzionale secondo un fattore $1/\varepsilon_0$ "

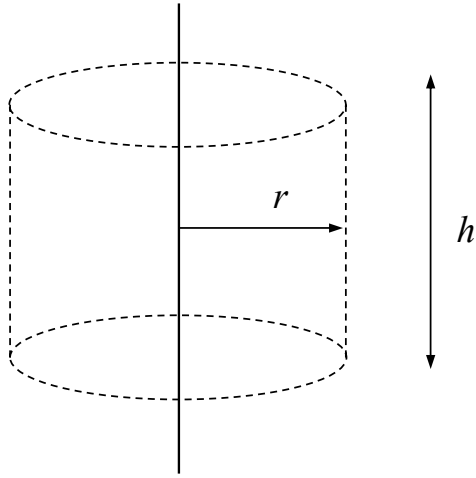
$$\Phi(\vec{E}) = \oint \vec{E} \cdot d\vec{S} = \frac{q_{int}}{\varepsilon_0}$$

sempre valido, non sempre utile



Teorema di Gauss

filo rettilineo infinito unif. carico



per simmetria, il campo elettrico è

- radiale rispetto al filo
- invariante per traslazione lungo il filo
- invariante per rotazione attorno al filo



simmetrica cilindrica

$$\Phi(\vec{E}) = \underbrace{\oint \vec{E} \cdot d\vec{S}}_{\text{flusso}} = \underbrace{\frac{q_{int}}{\epsilon_0}}_{\text{Gauss}}$$

$$\Phi(\vec{E}) = \oint \vec{E} \cdot d\vec{S} = \int_{\text{lato}} \vec{E} \cdot d\vec{S} + \cancel{\int_{\text{basi}} \vec{E} \cdot d\vec{S}} =$$

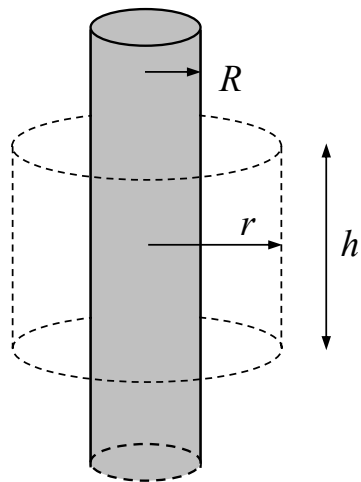
$$= \int_{\text{lato}} E dS = E \int_{\text{lato}} dS = E 2\pi r h =$$

$$= \frac{q_{int}}{\epsilon_0} = \frac{\lambda h}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0} \frac{1}{r}$$

Teorema di Gauss

cilindro rettilineo infinito unif. carico



per simmetria, il campo elettrico è

- radiale rispetto al filo
- invariante per traslazione lungo il filo
- invariante per rotazione attorno al filo

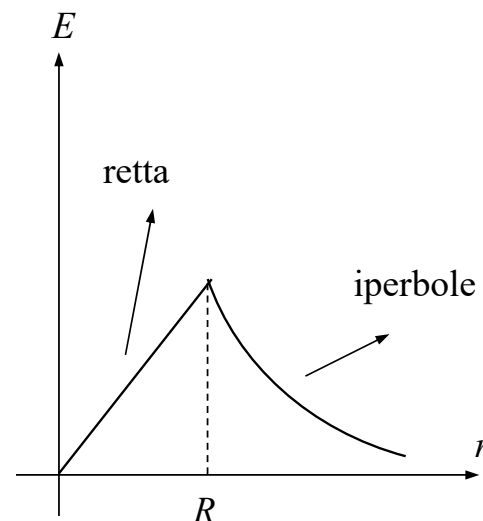


simmetrica cilindrica

$$\Phi(\vec{E}) = \oint \vec{E} \cdot d\vec{S} = E 2\pi r h = \frac{q_{int}}{\epsilon_0}$$

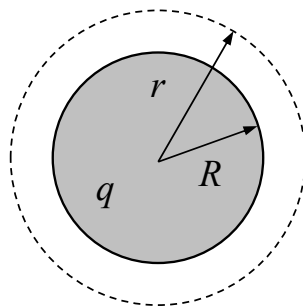
$$r > R: \quad q_{int} = \rho V = \rho \pi R^2 h \quad E = \frac{\rho R^2}{2\epsilon_0} \frac{1}{r}$$

$$r < R: \quad q_{int} = \rho V = \rho \pi r^2 h \quad E = \frac{\rho}{2\epsilon_0} r$$



Teorema di Gauss

sfera unif. carica



$$\Phi(\vec{E}) = \oint \vec{E} \cdot d\vec{S} = E 4\pi r^2 = \frac{q_{int}}{\epsilon_0}$$

$$r > R: \quad q_{int} = q \quad E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

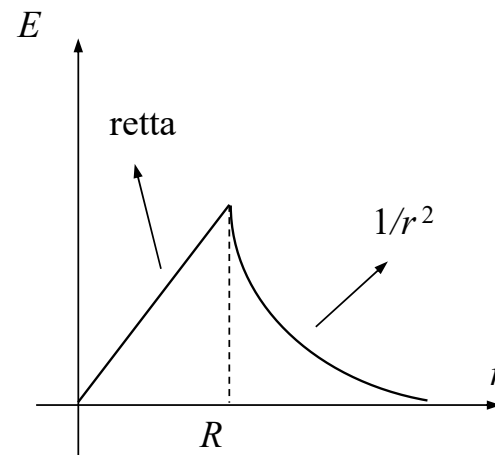
$$r < R: \quad q_{int} = \rho V = \frac{q}{\frac{4}{3}\pi R^3} \frac{4}{3}\pi r^3 = \frac{q}{R^3} r^3 \quad E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^3} r$$

per simmetria, il campo elettrico è

- radiale rispetto al centro
- invariante per rotazione della sfera



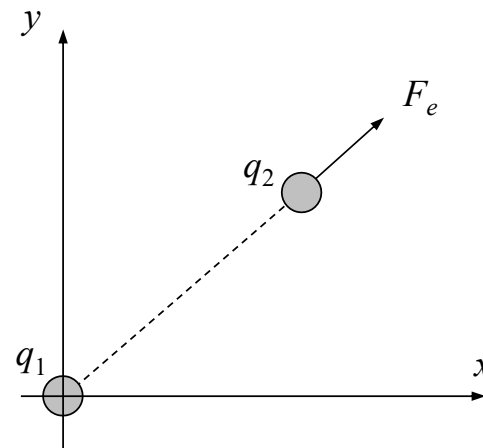
simmetrica sferica



Campo conservativo: energia potenziale

$$\vec{F}_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \vec{u}_r \quad d\vec{r} = dr \vec{u}_r + r d\theta \vec{u}_\theta$$

$$W = \int_A^B \vec{F}_e \cdot d\vec{r} = \int_{r_A}^{r_B} \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} dr$$



$$U_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

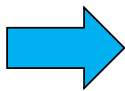
**energia potenziale
della forza elettrica**

$$W = \int_{r_A}^{r_B} \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} dr = - \int_A^B d \left(\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} \right) = - \int_A^B dU_e = -\Delta U_e$$

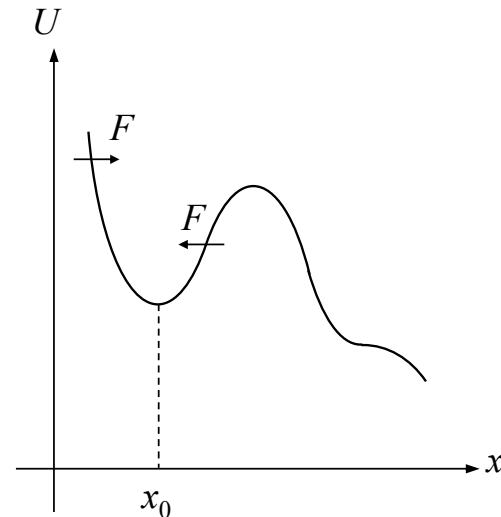


Campo conservativo: energia potenziale

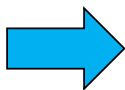
$$U(\vec{r}) = U_0 - \int_{\vec{r}_0}^{\vec{r}} \vec{F} \cdot d\vec{r}$$



$$\vec{F} = ?$$



$$\left\{ \begin{array}{l} dW = \vec{F} \cdot d\vec{r} = F_x dx + F_y dy + F_z dz = -dU \\ dU = \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy + \frac{\partial U}{\partial z} dz \end{array} \right.$$



$$\left\{ \begin{array}{l} F_x = -\frac{\partial U}{\partial x} \\ F_y = -\frac{\partial U}{\partial y} \\ F_z = -\frac{\partial U}{\partial z} \end{array} \right.$$

$$\vec{F} = -\text{grad}(U)$$

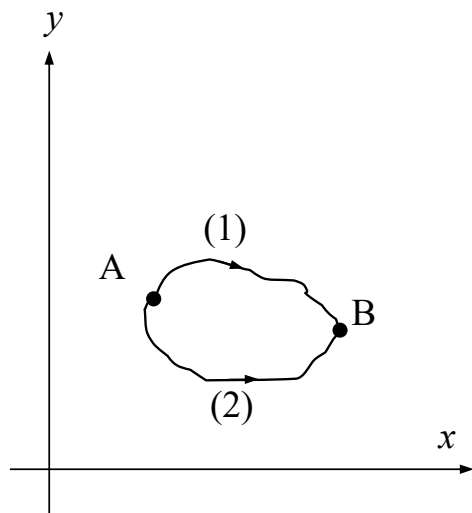
gradiente



Campo conservativo: energia potenziale

$$W = \int_{(1) A}^B \vec{F} \cdot d\vec{r} = \int_{(2) A}^B \vec{F} \cdot d\vec{r}$$

$$W = \oint \vec{F} \cdot d\vec{r} = \int_{(1) A}^B \vec{F} \cdot d\vec{r} + \int_{(2) B}^A \vec{F} \cdot d\vec{r} = \int_{(1) A}^B \vec{F} \cdot d\vec{r} - \int_{(2) A}^B \vec{F} \cdot d\vec{r} = 0$$



$$\vec{F} = q\vec{E}$$
$$\Lambda(\vec{F}) = \oint \vec{F} \cdot d\vec{r} = q \oint \vec{E} \cdot d\vec{r} = 0$$

circuitazione



III legge di Maxwell

"La circuitazione del campo elettrico lungo una linea chiusa
è nulla"

$$\Lambda(\vec{E}) = \oint \vec{E} \cdot d\vec{r} = 0$$



Campo conservativo: potenziale elettrico

$$U_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$[V] = \frac{[U]}{[q]} = \frac{\text{J}}{\text{C}} = \text{V}$$

volt

$$U_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} = q_1 \left(\frac{1}{4\pi\epsilon_0} \frac{q_2}{r} \right) = q_1 V_2$$

effetto

causa

oggetto

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

potenziale elettrico

$$U_I = \sum U_i = \sum q_I V_i = q_I \sum V_i = q_I V$$

vale la sovrapposizione degli effetti

$$V = \sum V_i$$

$$V = \int dV$$



Campo conservativo: potenziale elettrico

$$\begin{array}{ccc} & \vec{F} = -\text{grad}(U) & \\ \nearrow & & \nwarrow \\ \vec{F} = q\vec{E} & \Downarrow & U = qV \end{array}$$

$$\vec{E} = -\text{grad}(V)$$

$$[E] = \frac{[F]}{[q]} = \frac{\text{N}}{\text{C}} = \frac{\text{V}}{\text{m}}$$

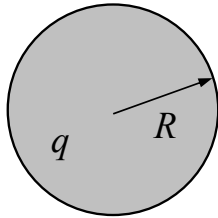
$$\left\{ \begin{array}{l} F_x = -\frac{\partial U}{\partial x} \\ F_y = -\frac{\partial U}{\partial y} \\ F_z = -\frac{\partial U}{\partial z} \end{array} \right.$$

$$\left\{ \begin{array}{l} E_x = -\frac{\partial V}{\partial x} \\ E_y = -\frac{\partial V}{\partial y} \\ E_z = -\frac{\partial V}{\partial z} \end{array} \right.$$



Campo conservativo: potenziale elettrico

sfera unif. carica



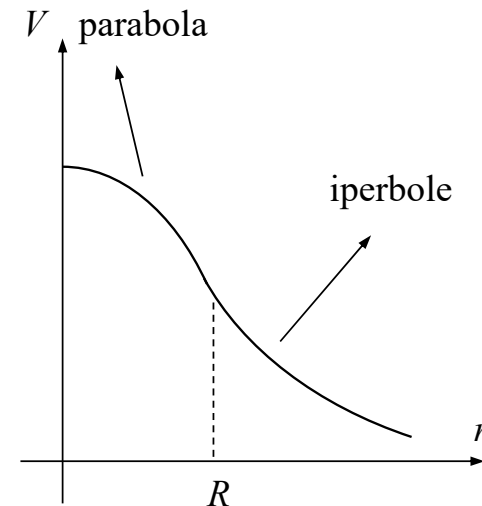
$$r > R: E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

$$r < R: E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^3} r$$

$$E_r = -\frac{\partial V}{\partial r}$$

$$r > R: V(r) = \cancel{V_\infty} - \int_{\textcircled{r_\infty}}^r E \, dr = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$r < R: V(r) = \textcircled{V_R} - \int_{\textcircled{R}}^r E \, dr = \frac{1}{8\pi\epsilon_0} \frac{q}{R} \left(3 - \frac{r^2}{R^2} \right)$$



$$U_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

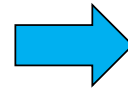
$$W_{int} = \int_A^B \vec{F} \cdot d\vec{r} = -\Delta U$$



$$W_{ext} = \int_A^B \vec{F}_{ext} \cdot d\vec{r} = \int_A^B -\vec{F} \cdot d\vec{r} = -W_{int} = \Delta U = U_f - U_i$$

$$\vec{r}_i = \infty$$

$$U_i = 0$$



$$W_{ext} = U_f$$

Campo conservativo: energia elettrica

$$W_1 = 0$$

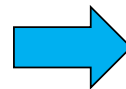
$$W_2 = \Delta U_2 = q_2 \Delta V = q_2 V_1 = q_2 \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{12}}$$

$$W_3 = \Delta U_3 = q_3 \Delta V = q_3 (V_1 + V_2) = q_3 \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}} \right)$$

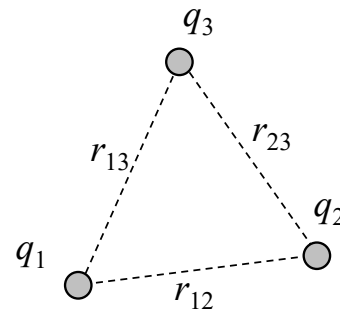
$$E_e = W = W_1 + W_2 + W_3 = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

$$E_e = \frac{1}{2} \sum_{i \neq j}^n U_{ij}$$

$$E_e = \frac{1}{2} \sum_{i=1}^n q_i V_i$$



$$U_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$



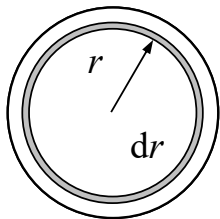
energia elettrica

$$E_e = \frac{1}{2} \int V \, dq$$



Campo conservativo: energia elettrica

sfera unif. carica



$$r > R: \quad V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$r < R: \quad V = \frac{1}{8\pi\epsilon_0} \frac{q}{R} \left(3 - \frac{r^2}{R^2} \right)$$

$$E_e = \frac{1}{2} \int V dq = \frac{1}{2} \int_0^R \frac{1}{8\pi\epsilon_0} \frac{q}{R} \left(3 - \frac{r^2}{R^2} \right) \rho \, 4\pi r^2 dr = \frac{3q^2}{20\pi\epsilon_0 R}$$

$$dq = \rho \, dV = \rho \, 4\pi r^2 dr$$

$$\rho = \frac{q}{\frac{4}{3}\pi R^3}$$

e se tutta la carica
andasse sulla superficie?

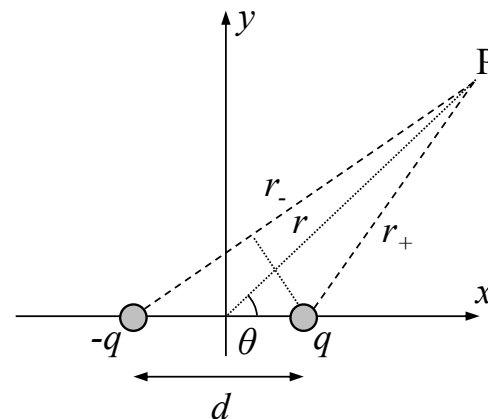
$$\begin{aligned} E_e &= \frac{1}{2} V q = \\ &= \frac{1}{2} \frac{q}{4\pi\epsilon_0 R} q = \frac{q^2}{8\pi\epsilon_0 R} \end{aligned}$$



Dipolo elettrico: interazioni create

$$V_+ = \frac{1}{4\pi\epsilon_0} \frac{q}{r_+} \quad V_- = \frac{1}{4\pi\epsilon_0} \frac{-q}{r_-}$$

$$V = V_+ + V_- = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_+} + \frac{-1}{r_-} \right) = \frac{q}{4\pi\epsilon_0} \frac{r_- - r_+}{r_- r_+}$$



approssimazione di dipolo

$$r \gg d \quad \begin{cases} r_- - r_+ \approx d \cos(\theta) \\ r_- r_+ \approx r^2 \end{cases}$$

$$\vec{p} = q\vec{d} \quad [p] = [q][d] = \text{Cm}$$

momento di dipolo elettrico

$$V = \frac{q}{4\pi\epsilon_0} \frac{r_- - r_+}{r_- r_+} \approx \frac{1}{4\pi\epsilon_0} \frac{qd \cos(\theta)}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{p \cos(\theta)}{r^2}$$

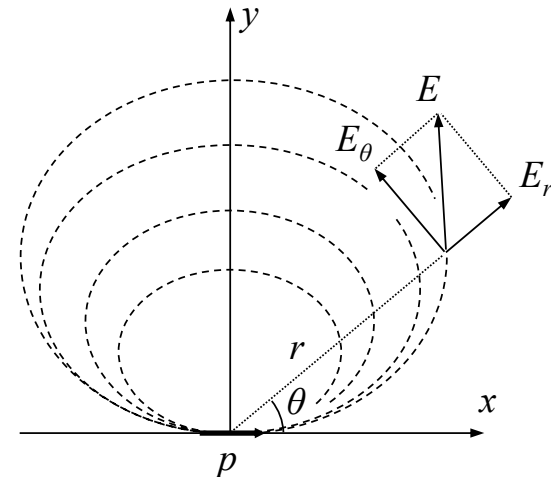
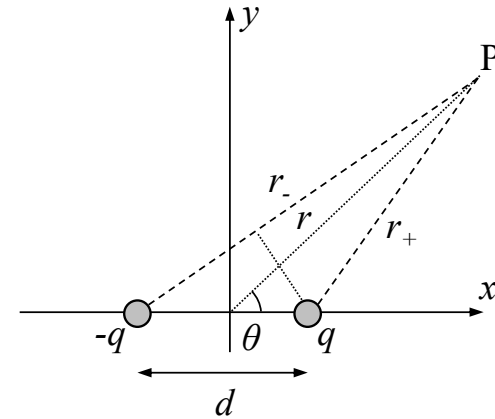


Dipolo elettrico: interazioni create

$$V = \frac{1}{4\pi\epsilon_0} \frac{p \cos(\theta)}{r^2}$$

$$\vec{E} = -\text{grad}(V) \quad \left\{ \begin{array}{l} E_r = -\frac{dV}{dr} = \frac{1}{4\pi\epsilon_0} \frac{2p \cos(\theta)}{r^3} \\ E_\theta = -\frac{1}{r} \frac{dV}{d\theta} = \frac{1}{4\pi\epsilon_0} \frac{p \sin(\theta)}{r^3} \end{array} \right.$$

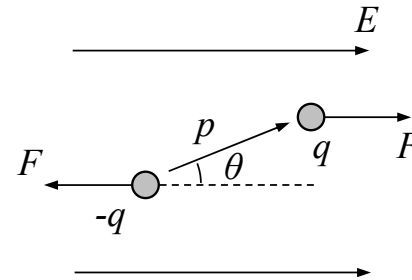
$$\left\{ \begin{array}{ll} \theta = 0: & E_\theta = 0 \quad E_r = \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3} \\ \theta = \frac{\pi}{2}: & E_\theta = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3} \quad E_r = 0 \end{array} \right.$$



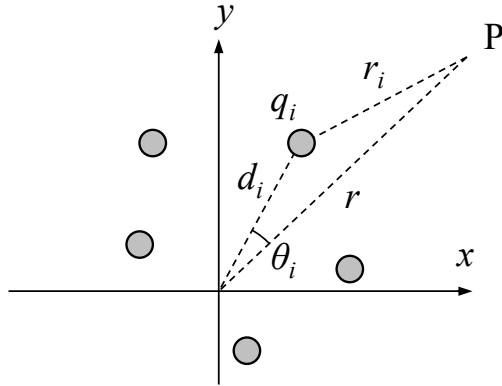
$$\vec{M} = \vec{d} \times \vec{F} = \vec{d} \times q\vec{E} = q\vec{d} \times \vec{E} = \vec{p} \times \vec{E}$$

$$\begin{aligned} U &= qV_+ - qV_- = q(V_+ - V_-) = q(-E \Delta x) = \\ &= q(-E d \cos(\theta)) = -pE \cos(\theta) = -\vec{p} \cdot \vec{E} \end{aligned}$$

$$\vec{F} = -\text{grad}(U) = \text{grad}(\vec{p} \cdot \vec{E})$$



Dipolo elettrico: sviluppo in multipoli



$$V_i = \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i}$$

$$\begin{aligned} r_i = |\vec{r} - \vec{d}_i| &= \sqrt{r^2 + d_i^2 - 2rd_i \cos(\theta_i)} = \\ &= r \sqrt{1 + \left(\frac{d_i}{r}\right)^2 - 2\frac{d_i}{r} \cos(\theta_i)} \end{aligned}$$

$$V_i = \frac{1}{4\pi\epsilon_0} \frac{q_i}{r \sqrt{1 + \left(\frac{d_i}{r}\right)^2 - 2\frac{d_i}{r} \cos(\theta_i)}} \approx$$

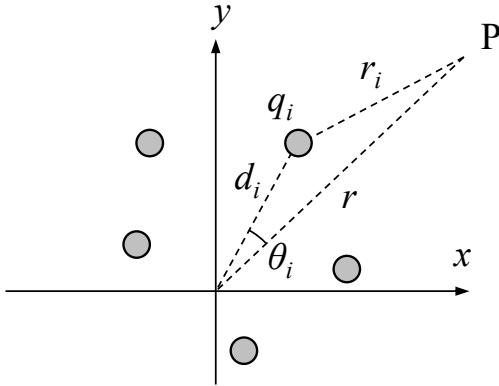
x

$$r \gg d_i : \frac{1}{\sqrt{1+x}} \approx 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \dots$$

$$\approx \frac{1}{4\pi\epsilon_0} \frac{q_i}{r} \left[1 + \cos(\theta_i) \left(\frac{d_i}{r}\right) + \frac{1}{2} \left(3 \cos^2(\theta_i) - 1 \right) \left(\frac{d_i}{r}\right)^2 + \dots \right] = \frac{1}{4\pi\epsilon_0} q_i \left[\frac{1}{r} + \frac{k_{i2}}{r^2} + \frac{k_{i3}}{r^3} + \dots \right]$$



Dipolo elettrico: sviluppo in multipoli



$$V_i = \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i} =$$

$$= \frac{1}{4\pi\epsilon_0} q_i \left(\frac{1}{r} + \frac{k_{i2}}{r^2} + \frac{k_{i3}}{r^3} + \dots \right)$$

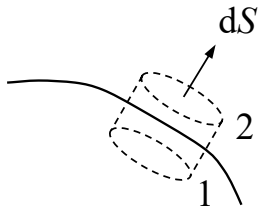
$$\begin{aligned} V &= \sum_i V_i = \frac{1}{4\pi\epsilon_0} \sum_i q_i \left(\frac{1}{r} + \frac{k_{i2}}{r^2} + \frac{k_{i3}}{r^3} + \dots \right) = \\ &= \frac{1}{4\pi\epsilon_0} \left[\frac{1}{r} \sum_i q_i + \frac{1}{r^2} \sum_i q_i k_{i2} + \frac{1}{r^3} \sum_i q_i k_{i3} + \dots \right] = \\ &= \frac{1}{4\pi\epsilon_0} \left(\frac{k_1}{r} + \frac{k_2}{r^2} + \frac{k_3}{r^3} + \dots \right) \end{aligned}$$

termine di...

$$\left\{ \begin{array}{ll} k_1 & \dots \text{monopolo} \\ k_2 & \dots \text{dipolo} \\ k_3 & \dots \text{quadrupolo} \end{array} \right.$$



Formulazione differenziale: condizioni al contorno



$$\left\{ \begin{array}{l} d\Phi_2(\vec{E}) = \vec{E}_2 \cdot d\vec{S}_2 = E_{n2} dS \\ d\Phi_1(\vec{E}) = \vec{E}_1 \cdot d\vec{S}_1 = -\vec{E}_1 \cdot d\vec{S}_2 = -E_{n1} dS \\ d\Phi_{lat}(\vec{E}) \approx 0 \end{array} \right.$$

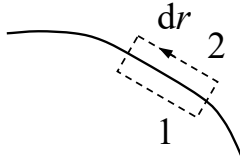
$$d\Phi(\vec{E}) = d\Phi_1(\vec{E}) + d\Phi_2(\vec{E}) + d\Phi_{lat}(\vec{E}) = (E_{n2} - E_{n1}) dS = \Delta E_n dS$$

$$d\Phi(\vec{E}) = \frac{dq}{\varepsilon_0} = \frac{\sigma dS}{\varepsilon_0}$$

$$\Delta E_n = \frac{\sigma}{\varepsilon_0}$$



Formulazione differenziale: condizioni al contorno



$$\left\{ \begin{array}{l} d\Lambda_2(\vec{E}) = \vec{E}_2 \cdot d\vec{r}_2 = E_{t2} dr \\ d\Lambda_1(\vec{E}) = \vec{E}_1 \cdot d\vec{r}_1 = -\vec{E}_1 \cdot d\vec{r}_2 = -E_{t1} dr \\ d\Lambda_n(\vec{E}) \approx 0 \end{array} \right.$$

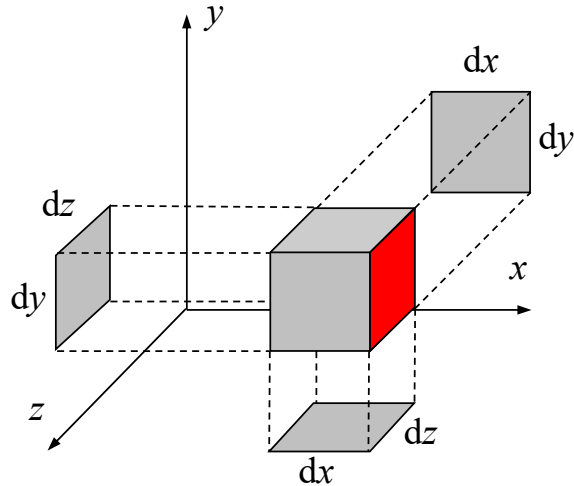
$$d\Lambda(\vec{E}) = d\Lambda_1(\vec{E}) + d\Lambda_2(\vec{E}) + d\Lambda_n(\vec{E}) = (E_{t2} - E_{t1}) dr = \Delta E_t dr$$

$$d\Lambda(\vec{E}) = 0$$

$$\Delta E_t = 0$$



Formulazione differenziale: leggi di Maxwell

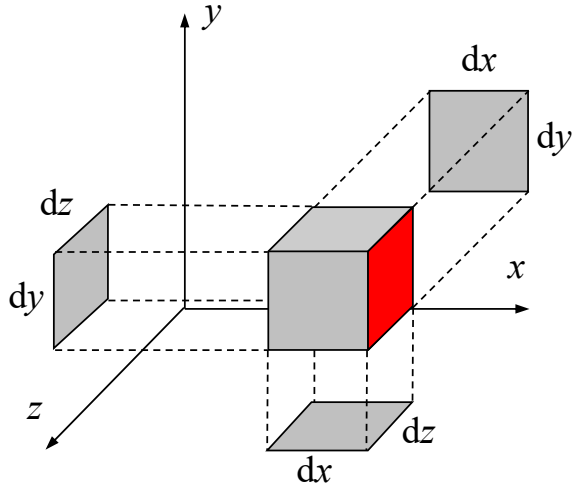


$$\left\{ \begin{array}{l} d\Phi_x''(\vec{E}) = \vec{E}'' \cdot d\vec{S} = E_x'' dS = E_x'' dydz \\ d\Phi_x'(\vec{E}) = \vec{E}' \cdot d\vec{S} = -E_x' dS = -E_x' dydz \end{array} \right.$$

$$\left\{ \begin{array}{l} d\Phi_x(\vec{E}) = d\Phi_x''(\vec{E}) + d\Phi_x'(\vec{E}) = E_x'' dydz - E_x' dydz = dE_x dydz = \left(\frac{\partial E_x}{\partial x} dx \right) dydz = \frac{\partial E_x}{\partial x} dV \\ d\Phi_y(\vec{E}) = \frac{\partial E_y}{\partial y} dV \\ d\Phi_z(\vec{E}) = \frac{\partial E_z}{\partial z} dV \end{array} \right.$$



Formulazione differenziale: leggi di Maxwell



$$\Phi(\vec{E}) = \int d\Phi(\vec{E}) = \oint \vec{E} \cdot d\vec{S} = \int \text{div}(\vec{E}) dV$$

teorema della divergenza

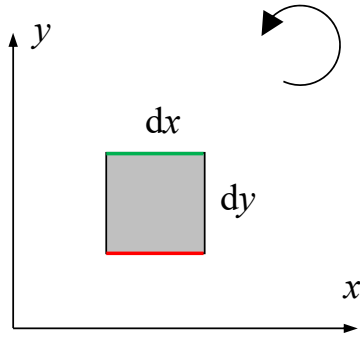
$$d\Phi(\vec{E}) = d\Phi_x(\vec{E}) + d\Phi_y(\vec{E}) + d\Phi_z(\vec{E}) = \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right) dV = \text{div}(\vec{E}) dV$$

$$d\Phi(\vec{E}) = \frac{dq}{\epsilon_0} = \frac{\rho dV}{\epsilon_0}$$

$$\text{div}(\vec{E}) = \frac{\rho}{\epsilon_0}$$



Formulazione differenziale: leggi di Maxwell



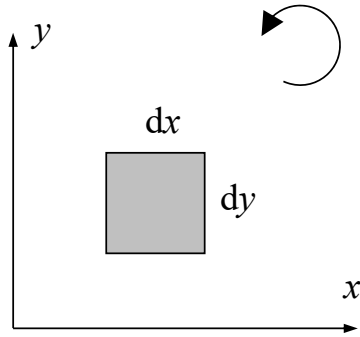
$$\left\{ \begin{array}{l} d\Lambda'_x(\vec{E}) = \vec{E}' \cdot d\vec{r} = E'_x dx \\ d\Lambda''_x(\vec{E}) = \vec{E}'' \cdot d\vec{r} = -E''_x dx \end{array} \right.$$

$$\left\{ \begin{array}{l} d\Lambda_x(\vec{E}) = d\Lambda'_x(\vec{E}) + d\Lambda''_x(\vec{E}) = E'_x dx - E''_x dx = -dE_x dx = -\left(\frac{\partial E_x}{\partial y} dy\right) dx \\ d\Lambda_y(\vec{E}) = dE_y dy = \frac{\partial E_y}{\partial x} dx dy \end{array} \right.$$

$$d\Lambda(\vec{E}) = d\Lambda_y(\vec{E}) + d\Lambda_x(\vec{E}) = \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) dx dy$$

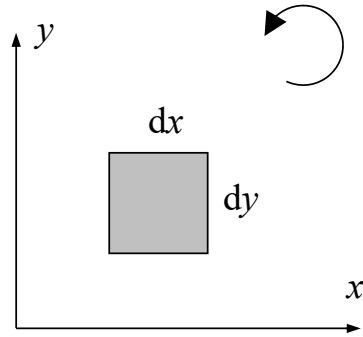


Formulazione differenziale: leggi di Maxwell



$$\left\{ \begin{array}{l} \text{piano } xy: \quad d\Lambda(\vec{E}) = \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) dx dy = \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) dS_z \\ \text{piano } yz: \quad d\Lambda(\vec{E}) = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) dy dz = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) dS_x \\ \text{piano } zx: \quad d\Lambda(\vec{E}) = \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) dz dx = \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) dS_y \end{array} \right.$$

Formulazione differenziale: leggi di Maxwell



$$\Lambda(\vec{E}) = \int d\Lambda(\vec{E}) = \oint \vec{E} \cdot d\vec{r} = \int \text{rot}(\vec{E}) \cdot d\vec{S}$$

teorema del rotore

$$d\Lambda(\vec{E}) = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) dS_x + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) dS_y + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) dS_z = \text{rot}(\vec{E}) \cdot d\vec{S}$$

$$d\Lambda(\vec{E}) = 0$$

$$\text{rot}(\vec{E}) = 0$$



Formulazione differenziale: leggi di Maxwell

	Teorema di Gauss	Campo conservativo
relazioni integrali	$\Phi(\vec{E}) = \oint \vec{E} \cdot d\vec{S} = \frac{q_{int}}{\varepsilon_0}$	$\Lambda(\vec{E}) = \oint \vec{E} \cdot d\vec{r} = 0$
condizioni al contorno	$\Delta E_n = \frac{\sigma}{\varepsilon_0}$	$\Delta E_t = 0$
relazioni infinitesime	$\text{div}(\vec{E}) = \frac{\rho}{\varepsilon_0}$	$\text{rot}(\vec{E}) = 0$



Formulazione differenziale: leggi di Maxwell

$$\left[\begin{array}{l} \text{grad}(V) = \frac{\partial V}{\partial x} \vec{u}_x + \frac{\partial V}{\partial y} \vec{u}_y + \frac{\partial V}{\partial z} \vec{u}_z \\ \text{div}(\vec{E}) = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \\ \text{rot}(\vec{E}) = \begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} \end{array} \right] \quad \rightarrow \quad \left[\begin{array}{l} \text{grad}(V) = \vec{\nabla} V \\ \text{div}(\vec{E}) = \vec{\nabla} \cdot \vec{E} \\ \text{rot}(\vec{E}) = \vec{\nabla} \times \vec{E} \end{array} \right]$$

$$\vec{\nabla} = \frac{\partial}{\partial x} \vec{u}_x + \frac{\partial}{\partial y} \vec{u}_y + \frac{\partial}{\partial z} \vec{u}_z$$

nabla



Formulazione differenziale: identità vettoriali

$$\left\{ \begin{array}{l} \text{grad}(V) = \frac{\partial V}{\partial x} \vec{u}_x + \frac{\partial V}{\partial y} \vec{u}_y + \frac{\partial V}{\partial z} \vec{u}_z \\ \\ \text{div}(\vec{E}) = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \\ \\ \text{rot}(\vec{E}) = \begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} \end{array} \right.$$

derivate prime

$$\text{div}(\vec{A} \times \vec{B}) = \text{rot}(\vec{A}) \cdot \vec{B} - \vec{A} \cdot \text{rot}(\vec{B})$$

$$\text{div}(f\vec{A}) = f \text{div}(\vec{A}) + \vec{A} \cdot \text{grad}(f)$$

$$\left\{ \begin{array}{l} \text{rot}(\text{grad}(f)) \equiv 0 \\ \\ \text{div}(\text{rot}(\vec{A})) \equiv 0 \\ \\ \nabla^2(\vec{A}) \equiv \text{grad}(\text{div}(\vec{A})) - \text{rot}(\text{rot}(\vec{A})) \end{array} \right.$$

derivate seconde

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

teorema di Schwarz(t)z



Formulazione differenziale: leggi di Maxwell

$$\left\{ \begin{array}{l} \operatorname{div}(\vec{E}) = \frac{\rho}{\varepsilon_0} \\ \operatorname{rot}(\vec{E}) = 0 \end{array} \right. \quad \operatorname{rot}(\operatorname{grad}(f)) \equiv 0 \quad \Rightarrow \quad \vec{E} = -\operatorname{grad}(V)$$

$$V = ?$$

$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho}{r} dV$$

vale la sovrapposizione degli effetti



Formulazione differenziale: leggi di Maxwell

$$\left\{ \begin{array}{l} \operatorname{div}(\vec{E}) = \frac{\rho}{\varepsilon_0} \\ \operatorname{rot}(\vec{E}) = 0 \end{array} \right. \quad \operatorname{rot}(\operatorname{grad}(f)) \equiv 0 \quad \Rightarrow \quad \vec{E} = -\operatorname{grad}(V)$$

arbitraria



$$V = V_0 + k$$

$$\begin{aligned} \operatorname{grad}(V) &= \operatorname{grad}(V_0 + k) = \\ &= \operatorname{grad}(V_0) + \cancel{\operatorname{grad}(k)} = \operatorname{grad}(V_0) \end{aligned}$$

$$-\operatorname{div}(\vec{E}) = -\operatorname{div}(-\operatorname{grad}(V)) = \nabla^2(V) = -\frac{\rho}{\varepsilon_0}$$

equazione di Poisson

$$\nabla^2 = \vec{\nabla} \cdot \vec{\nabla} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

laplaciano



Formulazione differenziale: energia elettrica

$$\operatorname{div}(\vec{E}) = \frac{\rho}{\varepsilon_0}$$

$$\operatorname{div}(f\vec{A}) = f \operatorname{div}(\vec{A}) + \vec{A} \cdot \operatorname{grad}(f)$$

$$\vec{E} = -\operatorname{grad}(V)$$

$$\rho V = \varepsilon_0 V \operatorname{div}(\vec{E}) = \varepsilon_0 \operatorname{div}(V\vec{E}) - \varepsilon_0 \vec{E} \cdot \operatorname{grad}(V) = \varepsilon_0 \operatorname{div}(V\vec{E}) + \varepsilon_0 E^2$$

$$E_{el} = \frac{1}{2} \int \rho V \, d\tau = \frac{1}{2} \varepsilon_0 \int \operatorname{div}(V\vec{E}) \, d\tau + \frac{1}{2} \varepsilon_0 \int E^2 \, d\tau$$

$$E_{el} = \int \frac{1}{2} \varepsilon_0 E^2 \, d\tau$$

$$\int \operatorname{div}(V\vec{E}) \, d\tau = \int V\vec{E} \, dS$$

$$VE \propto \frac{1}{r} \frac{1}{r^2}$$

$$S \propto r^2$$

$$\rho_{el} = \frac{1}{2} \varepsilon_0 E^2$$

densità di energia elettrica

