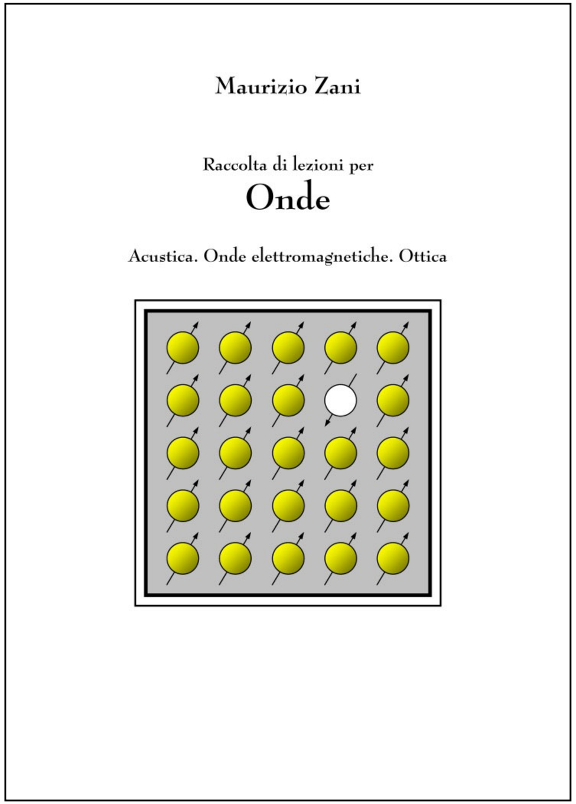
POLITECNICO
MILANO 1863

Onde

Acustica. Onde elettromagnetiche. Ottica

Maurizio Zani

Onde



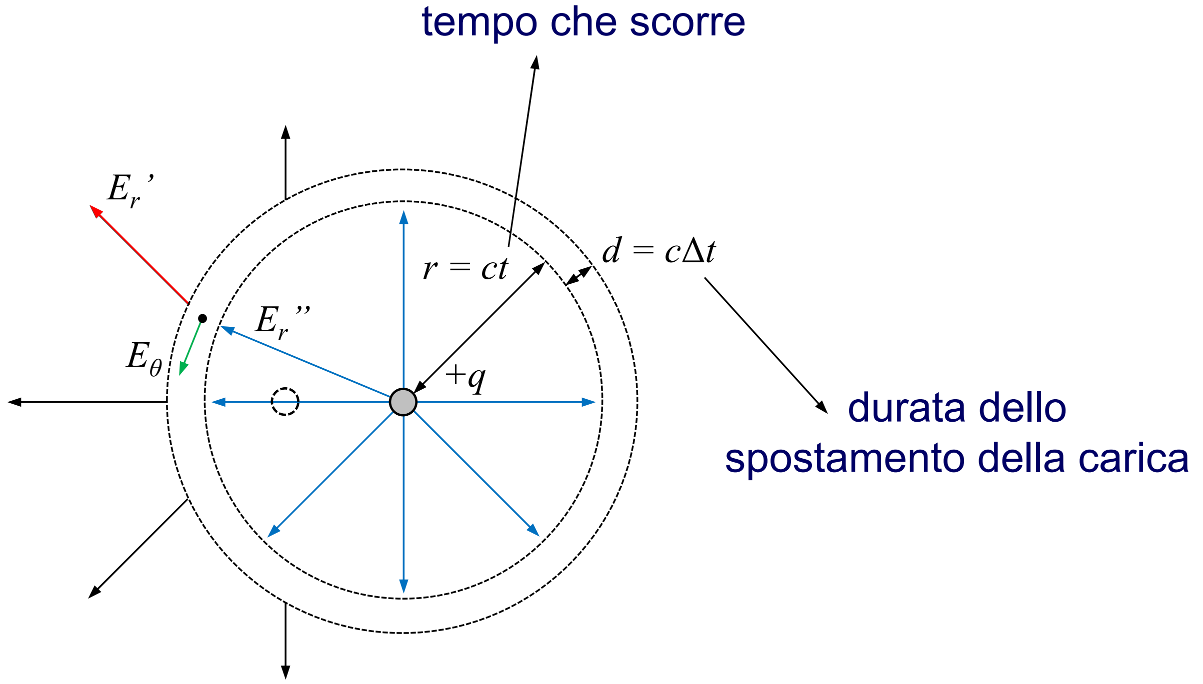
http://www.maurizio zani.it/wp/?p=2916

Onde

Emissione elettromagnetica
Interazione con la superficie
Modelli atomici
Interazione con la materia

Emissione elettromagnetica: carica accelerata

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\vec{u}_r$$



Emissione elettromagnetica: carica accelerata

decadenza $1/r^2$

carica ferma: $E_r = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

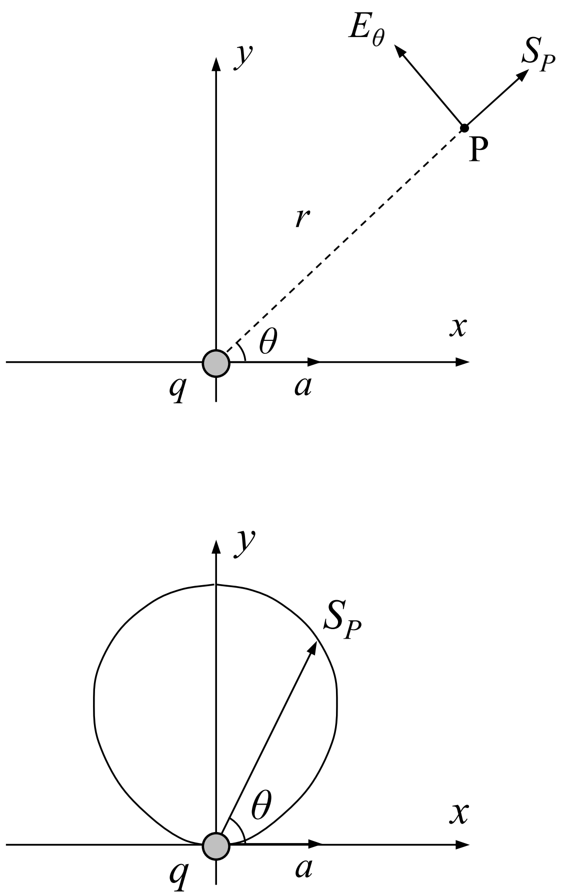
ritardo

carica accel.: $E_\theta(t) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\frac{a\overbrace{\left(t - r/c\right)}\sin(\theta)}{c^2}$

decadenza $1/r$

$$S_P = c\varepsilon_0 E^2 = \frac{1}{4\pi\varepsilon_0}\frac{1}{4\pi r^2}\frac{q^2 a^2 \sin^2(\theta)}{c^3}$$

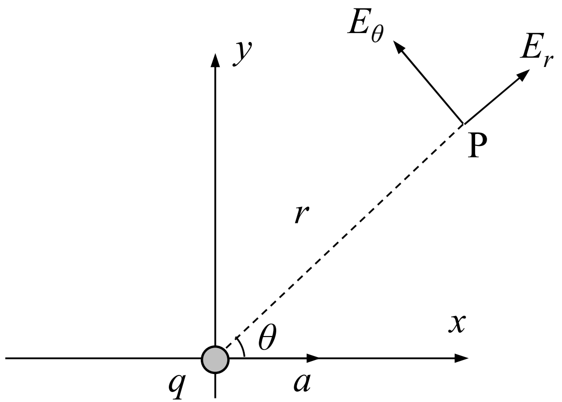
onda viaggiante



Emissione elettromagnetica: carica accelerata

$$S_P = c\varepsilon_0 E^2 = \frac{1}{4\pi\varepsilon_0}\frac{1}{4\pi r^2}\frac{q^2 a^2 \sin^2(\theta)}{c^3}$$

$$\mathrm{d}S = [r\,\mathrm{d}\theta]\left[\left(r\sin(\theta)\right)\mathrm{d}\varphi\right]$$

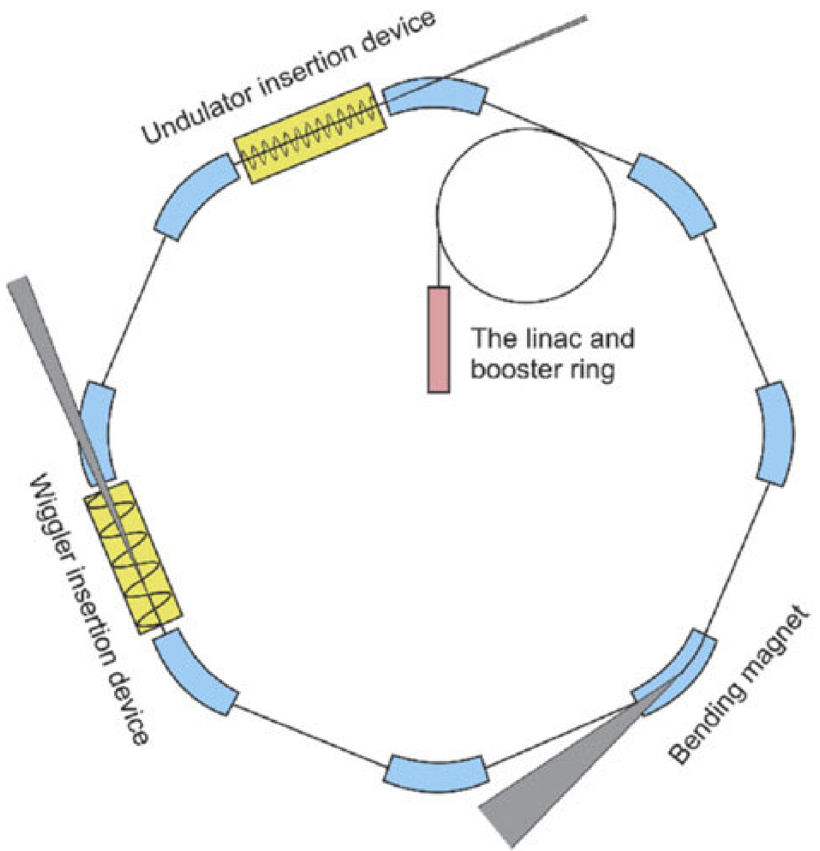


$$P_{rad} = \oint \vec{S}_P \cdot \mathrm{d}\vec{S} = \frac{1}{4\pi\varepsilon_0}\frac{1}{4\pi}\frac{q^2 a^2}{c^3}\int_0^{\pi} \sin^3(\theta)\,\mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\varphi = \frac{1}{6\pi\varepsilon_0}\frac{q^2 a^2}{c^3}$$

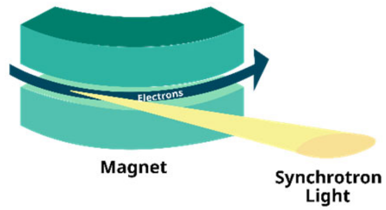
legge di Larmor

potenza irradiata dalla carica accelerata

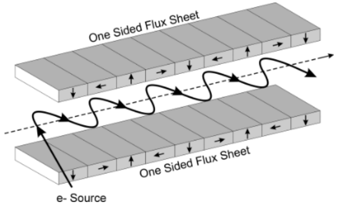
Emissione elettromagnetica: radiazione di sincrotrone



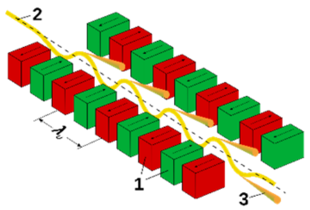
bending magnet



undulator

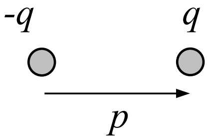


wiggler



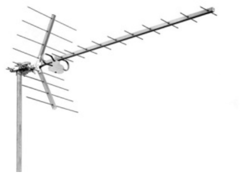
Emissione elettromagnetica: dipolo elettrico oscillante

$$x = x_0 \sin(\omega t) \qquad\qquad p = qx = qx_0 \sin(\omega t) = p_0 \sin(\omega t)$$



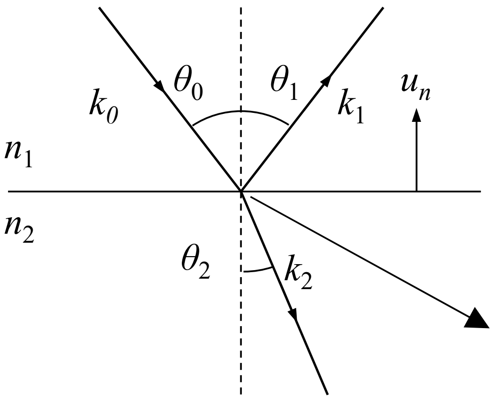
$$a = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\omega^2 x_0 \sin(\omega t) = -\frac{\omega^2}{q} p_0 \sin(\omega t)$$

$$P_{rad} = \frac{1}{6\pi\varepsilon_0}\frac{q^2 a^2}{c^3} = \frac{1}{6\pi\varepsilon_0}\frac{\omega^4 {p_0}^2 \sin^2(\omega t)}{c^3}$$



$$\langle P_{rad} \rangle = \left\langle \frac{1}{6\pi\varepsilon_0}\frac{{p_0}^2 \omega^4 \sin^2(\omega t)}{c^3} \right\rangle = \frac{1}{6\pi\varepsilon_0}\frac{{p_0}^2 \omega^4}{c^3}\left\langle \sin^2(\omega t) \right\rangle = \frac{1}{12\pi\varepsilon_0}\frac{{p_0}^2 \omega^4}{c^3}$$

Interazione con la superficie



piano di incidenza $\vec{k}_0 ; \vec{u}_n$

$$\vec{E}_0 = E_0 \sin\left(\vec{k}_0 \cdot \vec{r} - \omega_0 t + \varphi_0\right)\vec{u}_0$$ **onda incidente**

$$\vec{E}_1 = E_1 \sin\left(\vec{k}_1 \cdot \vec{r} - \omega_1 t + \varphi_1\right)\vec{u}_1$$ **onda riflessa**

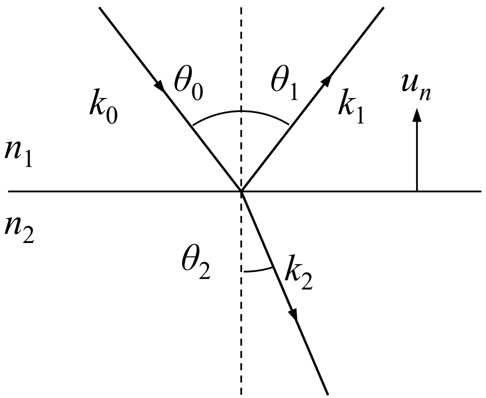
$$\vec{E}_2 = E_2 \sin\left(\vec{k}_2 \cdot \vec{r} - \omega_2 t + \varphi_2\right)\vec{u}_2$$ **onda trasmessa**

direzioni & intensità

leggi di Snell

leggi di Fresnel

Interazione con la superficie: leggi di Snell



condizioni al contorno ($\Delta E_t = 0$)

$$E_t = \left|\vec{u}_n \times \left(\vec{E}_0 + \vec{E}_1\right)\right| = \left|\vec{u}_n \times \vec{E}_2\right|$$

$$\left|\vec{u}_n \times \left[E_0 \sin\left(\vec{k}_0 \cdot \vec{r} - \omega_0 t + \varphi_0\right)\vec{u}_0 + E_1 \sin\left(\vec{k}_1 \cdot \vec{r} - \omega_1 t + \varphi_1\right)\vec{u}_1\right]\right| = \left|\vec{u}_n \times E_2 \sin\left(\vec{k}_2 \cdot \vec{r} - \omega_2 t + \varphi_2\right)\vec{u}_2\right|$$

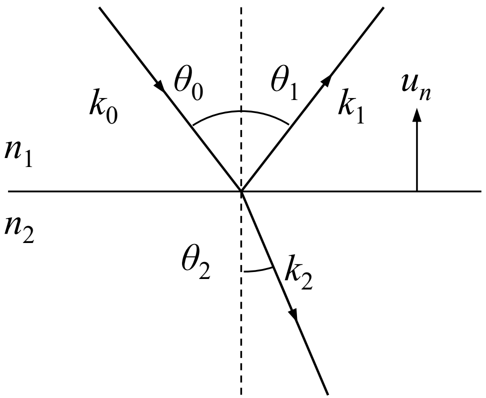
$\vec{r} = 0$

$$\left|\vec{u}_n \times \left[E_0 \sin\left(-\omega_0 t + \varphi_0\right)\vec{u}_0 + E_1 \sin\left(-\omega_1 t + \varphi_1\right)\vec{u}_1\right]\right| = \left|\vec{u}_n \times E_2 \sin\left(-\omega_2 t + \varphi_2\right)\vec{u}_2\right|$$

$\forall t$

$$\omega_0 = \omega_1 = \omega_2 = \omega$$

Interazione con la superficie: leggi di Snell



condizioni al contorno ($\Delta E_t = 0$)

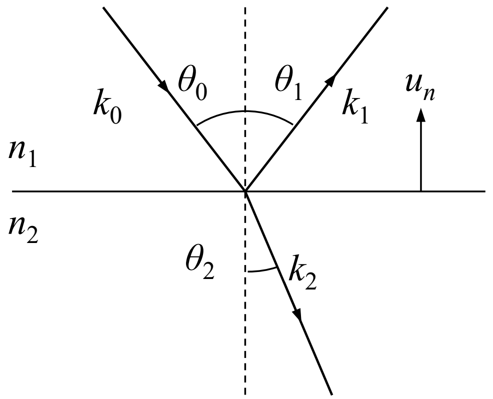
$$E_t = \left|\vec{u}_n \times \left(\vec{E}_0 + \vec{E}_1\right)\right| = \left|\vec{u}_n \times \vec{E}_2\right|$$

$$\left|\vec{u}_n \times \left[E_0 \sin\left(\vec{k}_0 \cdot \vec{r} - \omega t + \varphi_u\right)\vec{u}_0 + E_1 \sin\left(\vec{k}_1 \cdot \vec{r} - \omega t + \varphi_1\right)\vec{u}_1\right]\right| = \left|\vec{u}_n \times E_2 \sin\left(\vec{k}_2 \cdot \vec{r} - \omega t + \varphi_2\right)\vec{u}_2\right|$$

$\forall \vec{r} \in$ superf. $\qquad \vec{k}_0 \cdot \vec{r} - \cancel{\omega t} + \varphi_0 = \vec{k}_1 \cdot \vec{r} - \cancel{\omega t} + \varphi_1 = \vec{k}_2 \cdot \vec{r} - \cancel{\omega t} + \varphi_2$

$$\vec{k}_0 \cdot \vec{r} + \varphi_0 = \vec{k}_1 \cdot \vec{r} + \varphi_1 = \vec{k}_2 \cdot \vec{r} + \varphi_2$$

Interazione con la superficie: leggi di Snell



$$\underbrace{\vec{k}_0\cdot\vec{r}+\varphi_0 = \vec{k}_1\cdot\vec{r}+\varphi_1} = \vec{k}_2\cdot\vec{r}+\varphi_2$$

I e II termine

$$\left(\vec{k}_0 - \vec{k}_1\right)\cdot\vec{r} = \varphi_1 - \varphi_0 = \text{costante}$$

$\vec{r}_0;\vec{r}_1 \in$ superf.

$$\left(\vec{k}_0 - \vec{k}_1\right)\cdot\vec{r}_0 = \left(\vec{k}_0 - \vec{k}_1\right)\cdot\vec{r}_1$$

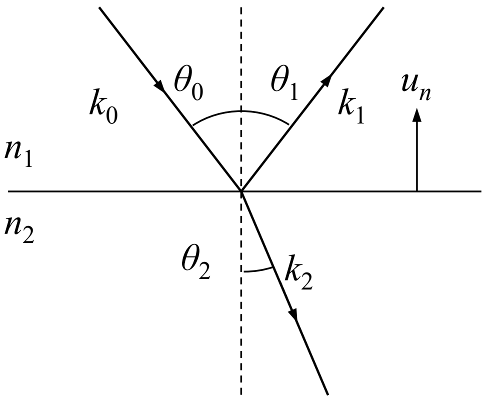
$$\left(\vec{k}_0 - \vec{k}_1\right)\cdot\left(\vec{r}_0 - \vec{r}_1\right) = 0$$

$\left(\vec{r}_0 - \vec{r}_1\right) \in$ superf.

$\left(\vec{k}_0 - \vec{k}_1\right) \perp$ superf.

$\vec{k}_1 \in$ piano inc.

Interazione con la superficie: leggi di Snell

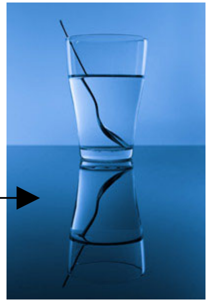


$$\underbrace{\vec{k}_0 \cdot \vec{r} + \varphi_0 = \vec{k}_1 \cdot \vec{r} + \varphi_1}_{\text{I e II termine}} = \vec{k}_2 \cdot \vec{r} + \varphi_2$$

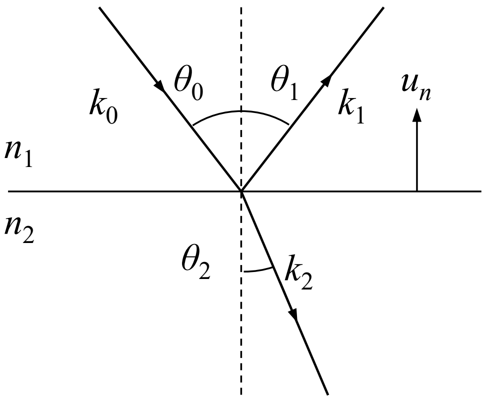
$$\left(\vec{k}_0 - \vec{k}_1\right) \cdot \vec{r} = \varphi_1 - \varphi_0 = \text{costante}$$

$$\left(\vec{k}_0 - \vec{k}_1\right) \times \vec{u}_n = 0 \qquad \vec{k}_0 \times \vec{u}_n = \vec{k}_1 \times \vec{u}_n \qquad k_0 \sin\left(\theta_0\right) = k_1 \sin\left(\theta_1\right)$$

$$k = \frac{\omega}{v} = \frac{\omega}{c} n \qquad n_1 \sin\left(\theta_0\right) = n_1 \sin\left(\theta_1\right) \qquad \theta_0 = \theta_1$$



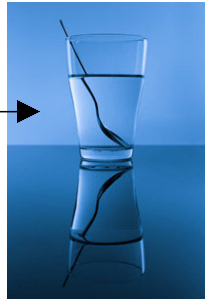
legge di Snell per la riflessione



$$\vec{k}_0 \cdot \vec{r} + \varphi_0 = \underbrace{\vec{k}_1 \cdot \vec{r} + \varphi_1 = \vec{k}_2 \cdot \vec{r} + \varphi_2}_{\text{II e III termine}}$$

$$\left(\vec{k}_1 - \vec{k}_2\right) \cdot \vec{r} = \varphi_2 - \varphi_1 = \text{costante}$$

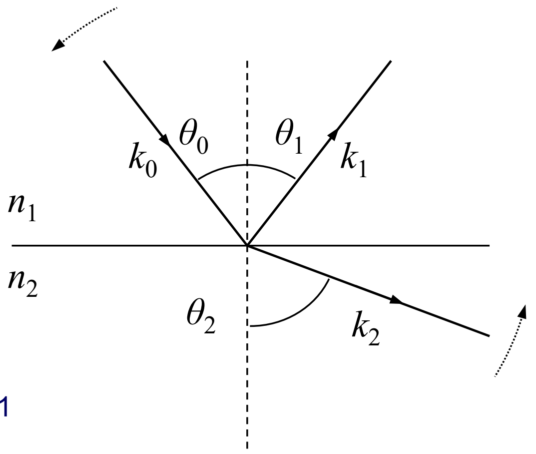
$$\left(\vec{k}_1 - \vec{k}_2\right) \times \vec{u}_n = 0 \qquad \vec{k}_1 \times \vec{u}_n = \vec{k}_2 \times \vec{u}_n \qquad k_1 \sin\left(\theta_1\right) = k_2 \sin\left(\theta_2\right)$$



$$k = \frac{\omega}{v} = \frac{\omega}{c} n \qquad n_1 \sin\left(\theta_1\right) = n_2 \sin\left(\theta_2\right)$$

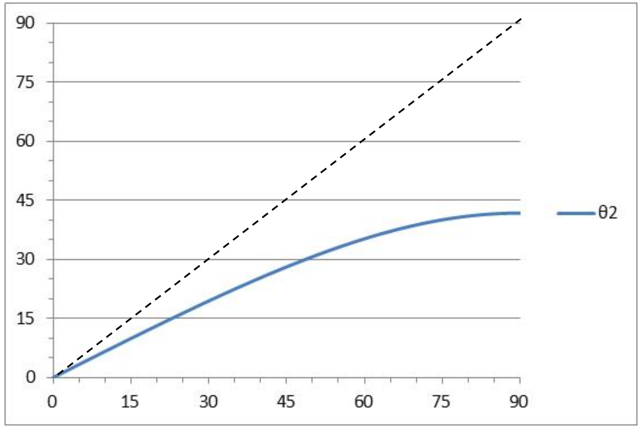
legge di Snell per la rifrazione

Interazione con la superficie: riflessione totale

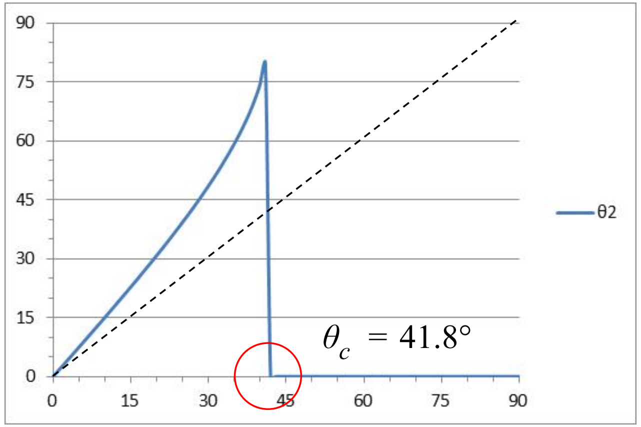


$n_2 < n_1$
$\theta_2 > \theta_1$

$n_2 > n_1$ ($n_1 = 1$; $n_2 = 1.5$)



$n_2 < n_1$ ($n_1 = 1.5$; $n_2 = 1$)

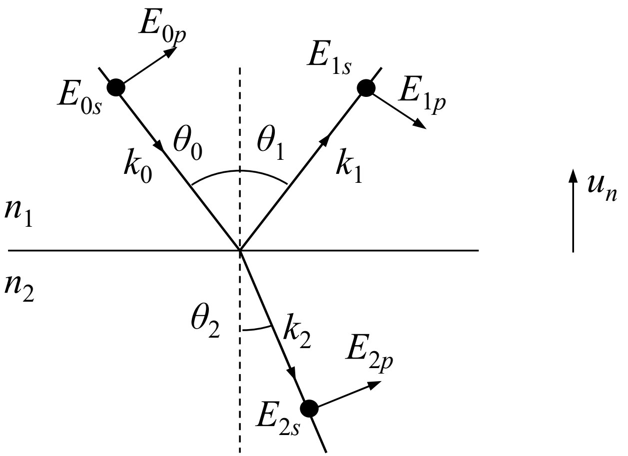


angolo critico (o limite)

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \longrightarrow \theta_2 = \frac{\pi}{2}$$

$$\theta_1 = \theta_c = \arcsin\left(\frac{n_2}{n_1}\right)$$

Interazione con la superficie



piano di incidenza $\vec{k}_0 ; \vec{u}_n$

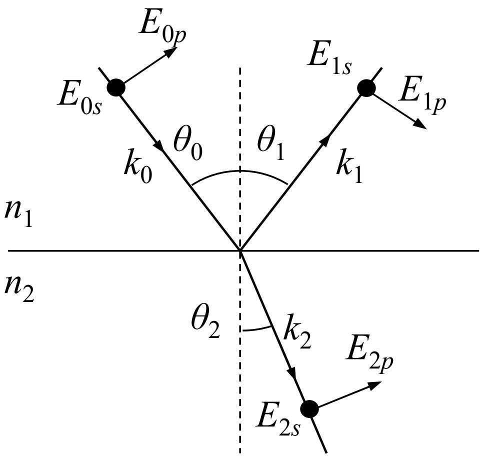
$$\vec{E}_0 = E_0 \sin\left(\vec{k}_0 \cdot \vec{r} - \omega_0 t + \varphi_0\right)\vec{u}_0 = \left(E_{0p}\vec{u}_p + E_{0s}\vec{u}_s\right)\sin\left(\vec{k}_0 \cdot \vec{r} - \omega_0 t + \varphi_0\right)$$

versore del campo elettrico

s: ortogonale al piano di incidenza

p: parallela al piano di incidenza

Interazione con la superficie: leggi di Fresnel



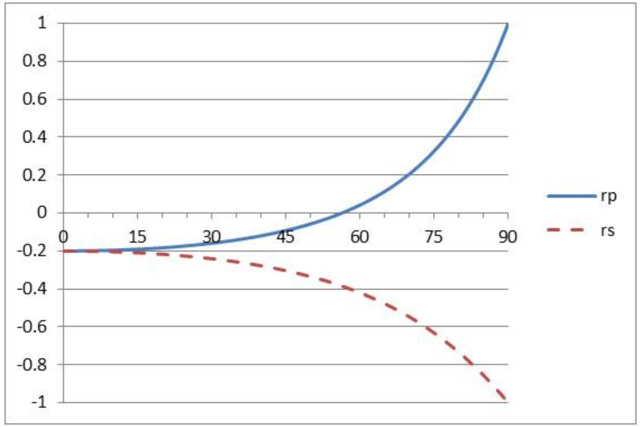
$$r_p = r_s = 0$$

coeff. di riflessione

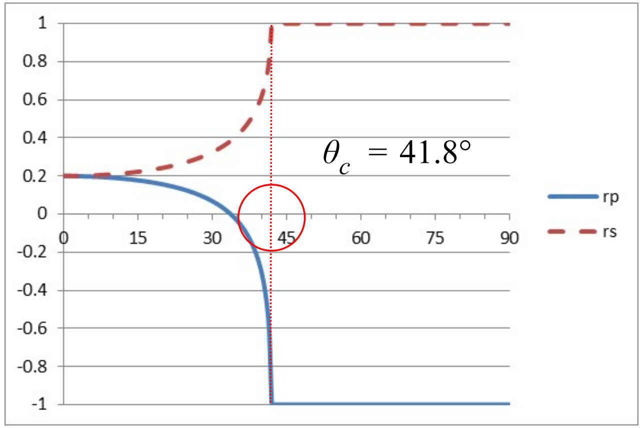
$$r_p = \frac{E_{1p}}{E_{0p}} = \frac{n_1 \cos(\theta_2) - n_2 \cos(\theta_1)}{n_1 \cos(\theta_2) + n_2 \cos(\theta_1)}$$

$$r_s = \frac{E_{1s}}{E_{0s}} = \frac{n_1 \cos(\theta_1) - n_2 \cos(\theta_2)}{n_1 \cos(\theta_1) + n_2 \cos(\theta_2)}$$

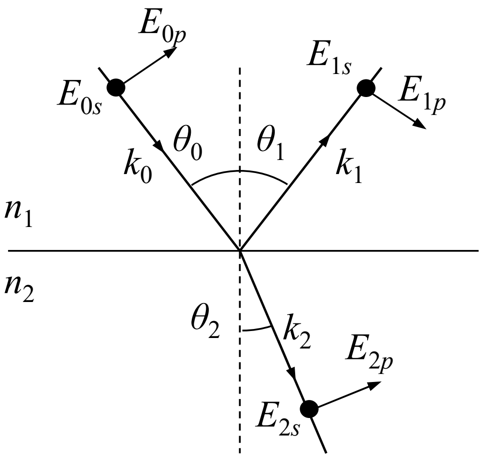
$n_2 > n_1$ ($n_1 = 1$; $n_2 = 1.5$)



$n_2 < n_1$ ($n_1 = 1.5$; $n_2 = 1$)

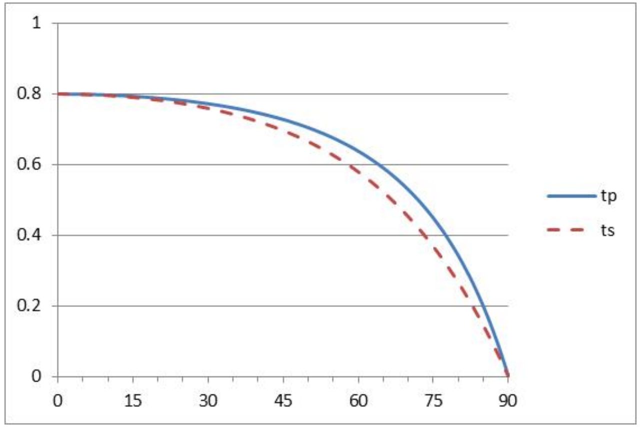


Interazione con la superficie: leggi di Fresnel

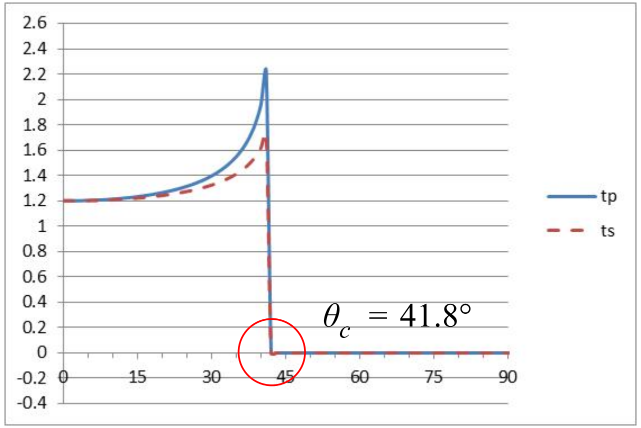


$$t_p = t_s = 0$$

$n_2 > n_1$ ($n_1 = 1$; $n_2 = 1.5$)



$n_2 < n_1$ ($n_1 = 1.5$; $n_2 = 1$)

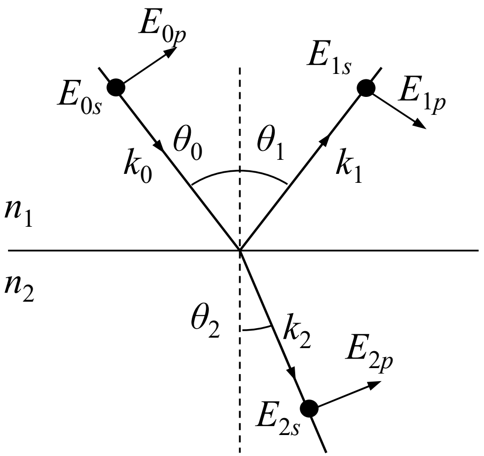


coeff. di trasmissione

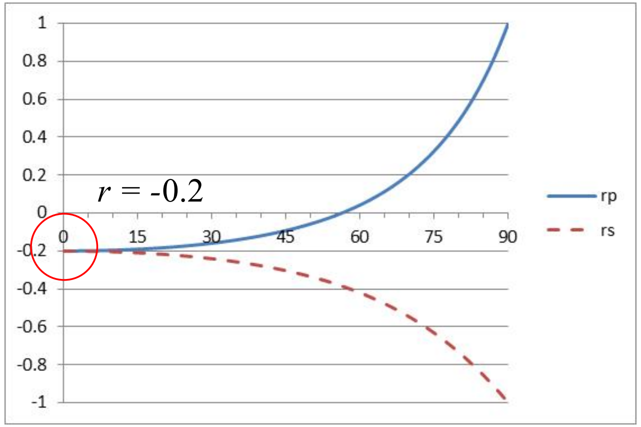
$$t_p = \frac{E_{2p}}{E_{0p}} = \frac{2n_1 \cos(\theta_1)}{n_1 \cos(\theta_2) + n_2 \cos(\theta_1)}$$

$$t_s = \frac{E_{2s}}{E_{0s}} = \frac{2n_1 \cos(\theta_1)}{n_1 \cos(\theta_1) + n_2 \cos(\theta_2)}$$

Interazione con la superficie: leggi di Fresnel



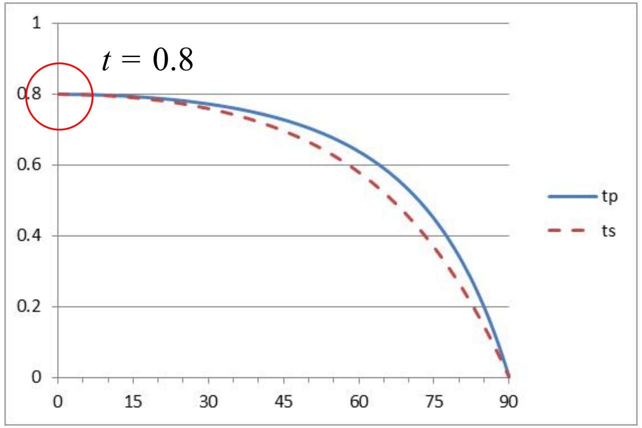
$n_2 > n_1$ ($n_1 = 1$; $n_2 = 1.5$)



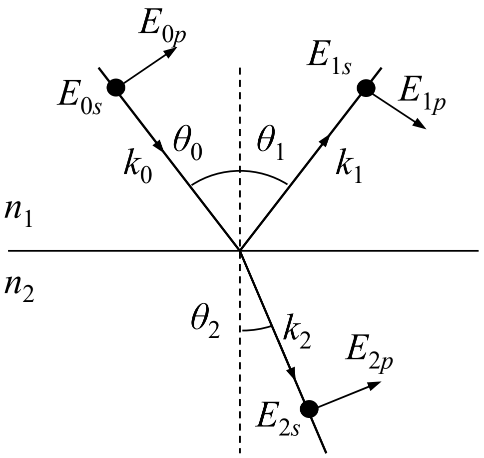
incidenza normale

$$r_p = r_s = \frac{n_1 - n_2}{n_1 + n_2}$$

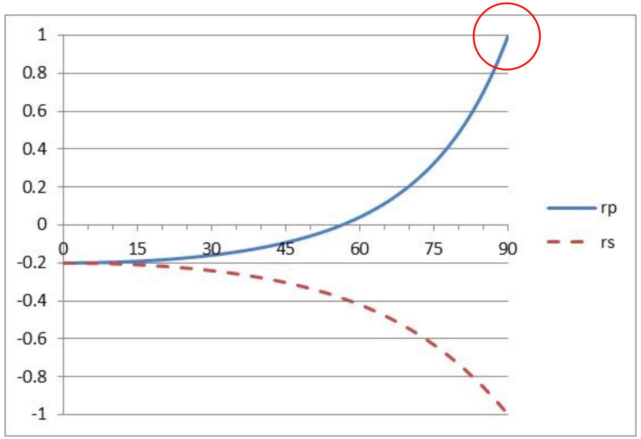
$$t_p = t_s = \frac{2n_1}{n_1 + n_2}$$



Interazione con la superficie: leggi di Fresnel



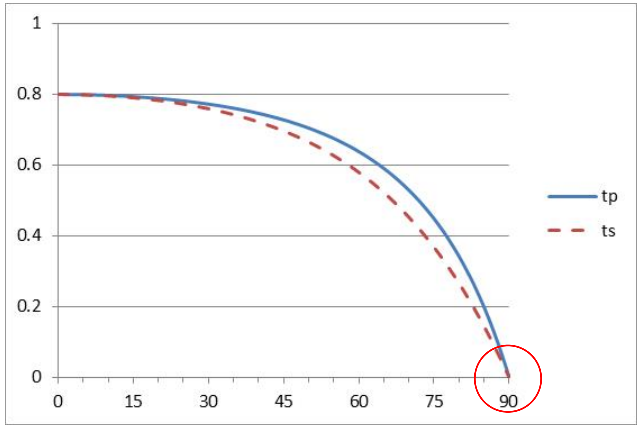
$n_2 > n_1$ ($n_1 = 1$; $n_2 = 1.5$)



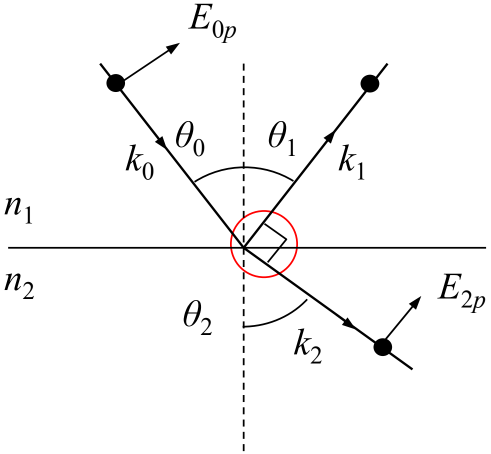
incidenza radente

$$r_p = -r_s = 1$$

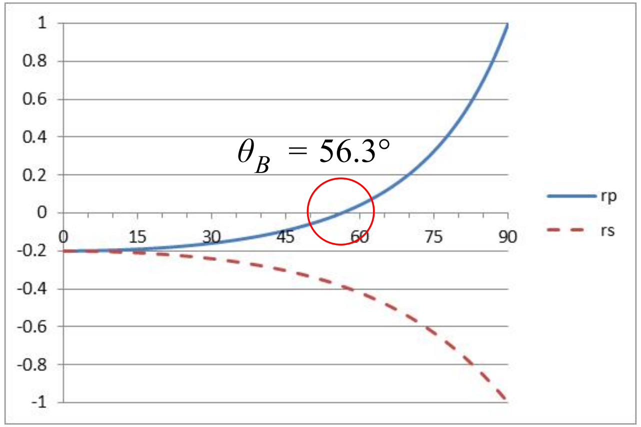
$$t_p = t_s \approx 0$$



Interazione con la superficie: rifrazione totale



$n_2 > n_1$ ($n_1 = 1$; $n_2 = 1.5$)

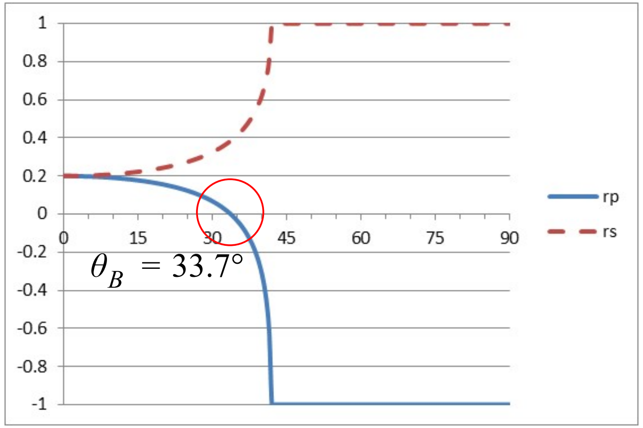


angolo di Brewster

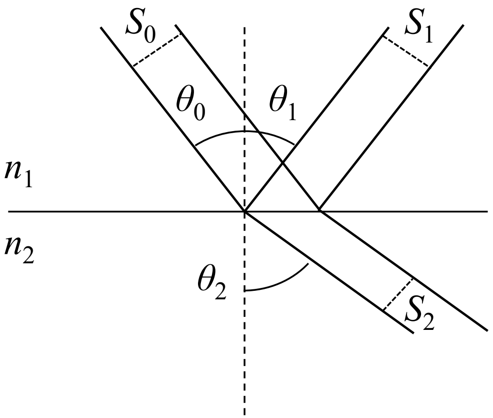
$$r_p = \frac{n_1 \cos(\theta_2) - n_2 \cos(\theta_1)}{n_1 \cos(\theta_2) + n_2 \cos(\theta_1)} = \frac{\tan(\theta_2 - \theta_1)}{\tan(\theta_2 + \theta_1)} = 0$$

$$\theta_1 = \theta_B = \text{arc tan}\left(\frac{n_2}{n_1}\right) \qquad \theta_2 + \theta_1 = \frac{\pi}{2}$$

$n_2 < n_1$ ($n_1 = 1.5$; $n_2 = 1$)



Interazione con la superficie: riflettanza e trasmittanza

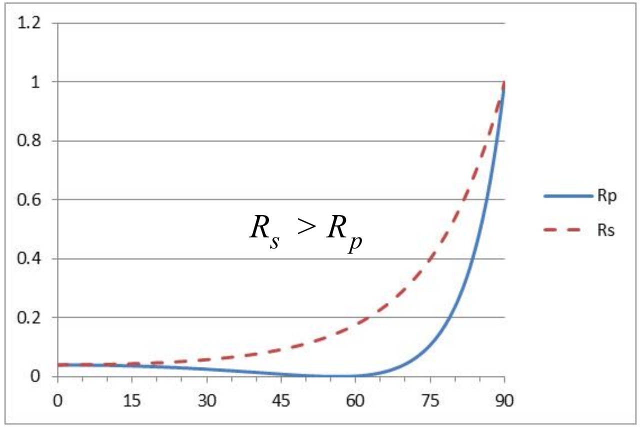


riflettanza

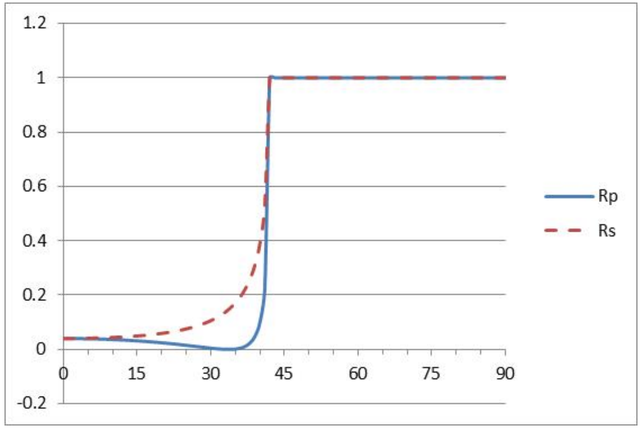
$$R = \frac{P_1}{P_0} = \frac{I_1 S_1}{I_0 S_0} = \frac{I_1}{I_0} = \left(\frac{E_1}{E_0}\right)^2 = r^2$$

$$I = n \cdot \frac{1}{2} c \varepsilon_0 E_0{}^2$$

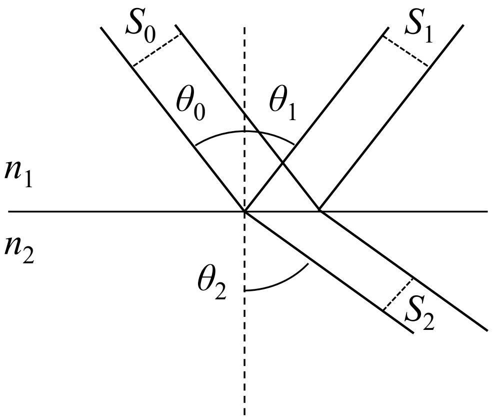
$n_2 > n_1$ ($n_1 = 1$; $n_2 = 1.5$)



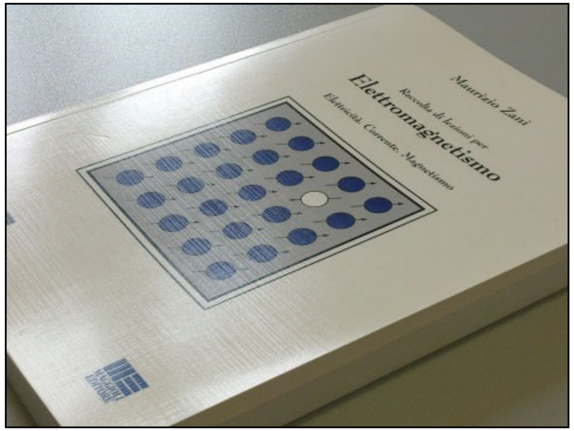
$n_2 < n_1$ ($n_1 = 1.5$; $n_2 = 1$)



Interazione con la superficie: riflettanza e trasmittanza



naturale



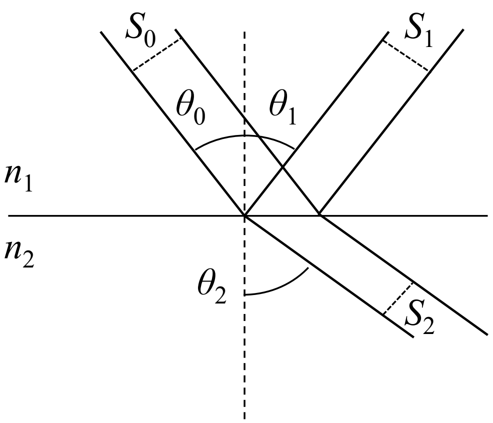
con polarizzatore verticale

riflettanza

$$R = \frac{P_1}{P_0} = \frac{I_1 S_1}{I_0 S_0} = \frac{I_1}{I_0} = \left(\frac{E_1}{E_0}\right)^2 = r^2$$

$$I = n \cdot \frac{1}{2} c \varepsilon_0 E_0{}^2$$

Interazione con la superficie: riflettanza e trasmittanza

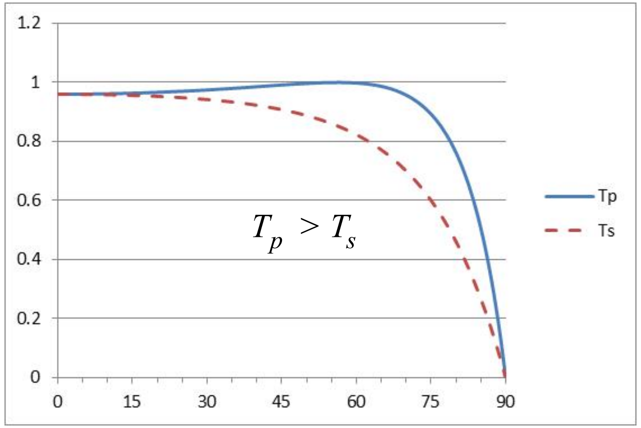


trasmittanza

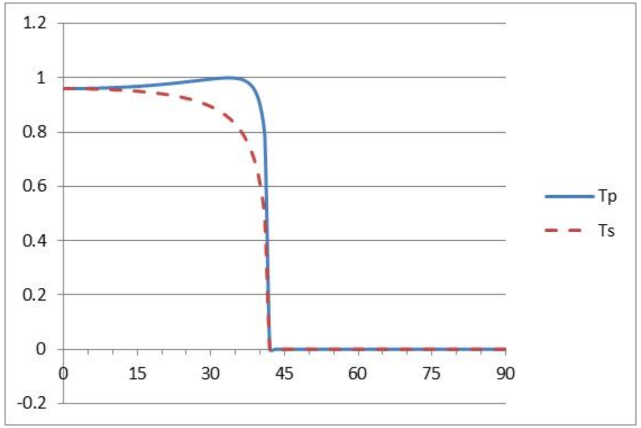
$$T = \frac{P_2}{P_0} = \frac{I_2 S_2}{I_0 S_0} = \frac{n_2 \cos(\theta_2)}{n_1 \cos(\theta_1)} t^2$$

$$I = n \cdot \frac{1}{2} c \varepsilon_0 E_0^{\,2}$$

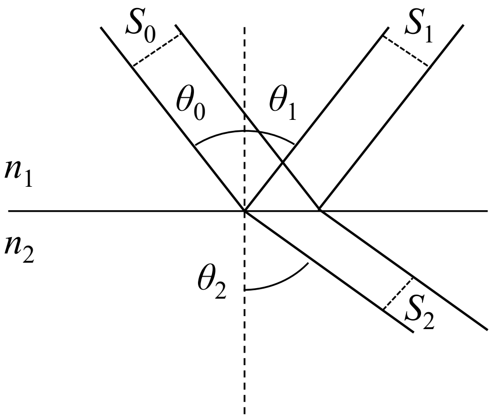
$n_2 > n_1$ ($n_1 = 1$; $n_2 = 1.5$)



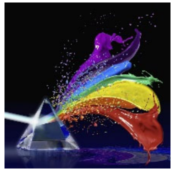
$n_2 < n_1$ ($n_1 = 1.5$; $n_2 = 1$)



Interazione con la superficie: conservazione dell'energia

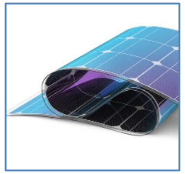


dispersione cromatica



&

assorbimento



riflettanza

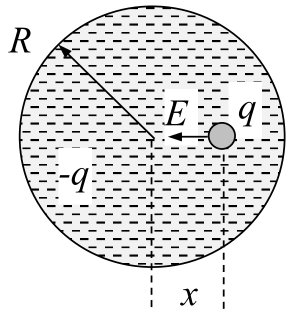
$$R + T + A = 1$$

trasmittanza

conservazione dell'energia

Modello atomico: modello di Lorentz

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{R^3} r\vec{u}_r$$



$$\vec{F} = q\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{R^3} x\vec{u}_x = -kx\vec{u}_x$$

moto armonico libero

pulsazione propria

$$x(t) = A\cos(\omega_0 t) \longrightarrow \omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{q^2}{4\pi\varepsilon_0 m R^3}}$$

Modello atomico: modello di Lorentz

$$F_{el} = -kx$$

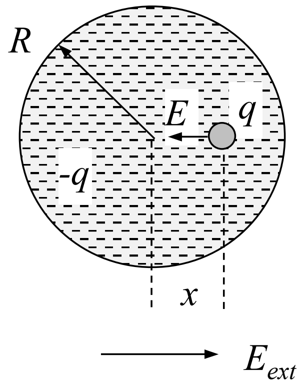
$$F_{vis} = -\beta\dot{x}$$

$$F_{ext}(t) = qE_{ext} = qE_0\cos(\omega t)$$

pulsazione forzante

moto armonico forzato

$$x(t) = A\cos(\omega t - \varphi)$$



Modello atomico: modello di Lorentz

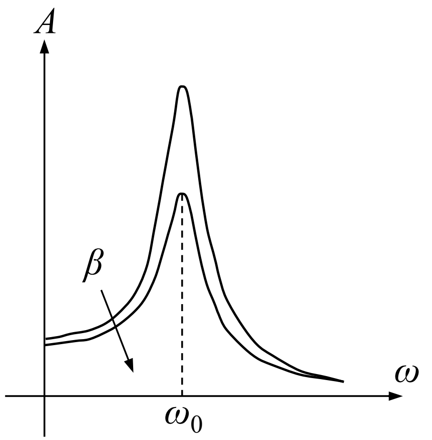
$$\omega_c = \frac{\beta}{2m}$$

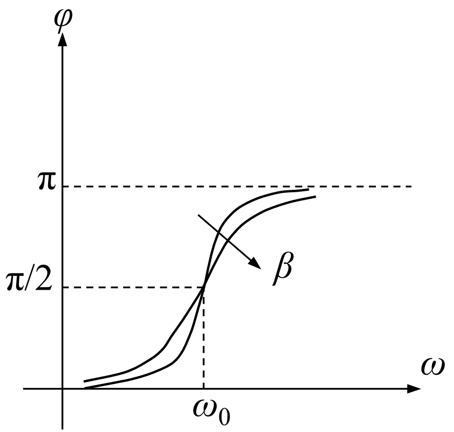
pulsazione critica

$$A = \frac{qE_0}{m} \frac{1}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + \left(2\omega_c\omega\right)^2}}$$

$$x(t) = A\cos\left(\omega t - \varphi\right)$$

$$\tan\left(\varphi\right) = \frac{2\omega_c\omega}{\omega_0^2 - \omega^2}$$



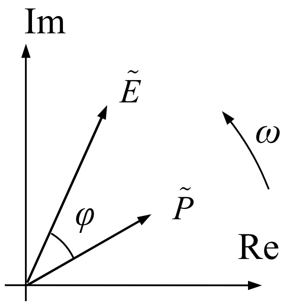


Modello atomico: permettività elettrica relativa

$$\begin{cases} E(t) = E_0 \cos(\omega t) \\ P(t) = nqx(t) = nqA\cos(\omega t - \varphi) \end{cases}$$

$$A = \frac{qE_0}{m} \frac{1}{\sqrt{\left(\omega_0^{\,2} - \omega^2\right)^2 + \left(2\omega_c \omega\right)^2}}$$

$$\tan(\varphi) = \frac{2\omega_c\omega}{\omega_0^{\,2} - \omega^2}$$



$$\begin{cases} \tilde{E} = E_0 \mathrm{e}^{\mathrm{i}\omega t} \\ \tilde{P} = nqA\mathrm{e}^{\mathrm{i}(\omega t - \varphi)} = nqA\mathrm{e}^{-\mathrm{i}\varphi} e^{\mathrm{i}\omega t} \end{cases}$$

$$\tilde{P} = \varepsilon_0 \left(\tilde{\varepsilon}_r - 1\right) \tilde{E}$$

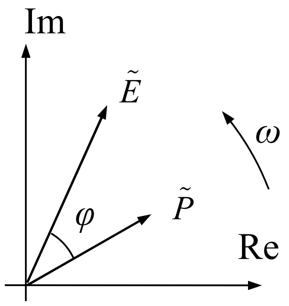
$$\tilde{\varepsilon}_r = 1 + \frac{\tilde{P}}{\varepsilon_0 \tilde{E}}$$

Modello atomico: permettività elettrica relativa

$$\begin{cases} \tilde{E} = E_0 e^{i\omega t} \\ \tilde{P} = nqAe^{i(\omega t - \varphi)} = nqAe^{-i\varphi}e^{i\omega t} \end{cases}$$

$$A = \frac{qE_0}{m}\frac{1}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + \left(2\omega_c\omega\right)^2}}$$

$$\tan(\varphi) = \frac{2\omega_c\omega}{\omega_0^2 - \omega^2}$$



$$\cos(\varphi) = \frac{1}{\sqrt{1 + \tan^2(\varphi)}}$$

$$e^{-i\varphi} = \cos(\varphi) - i\sin(\varphi)$$

$$\sin(\varphi) = \cos(\varphi)\tan(\varphi)$$

$$\tilde{\varepsilon}_r = 1 + \frac{\tilde{P}}{\varepsilon_0\tilde{E}} = 1 + \frac{nqAe^{-i\varphi}}{\varepsilon_0 E_0} = 1 + \omega_p^2\frac{\left(\omega_0^2 - \omega^2\right) - i\left(2\omega_c\omega\right)}{\left(\omega_0^2 - \omega^2\right)^2 + \left(2\omega_c\omega\right)^2}$$

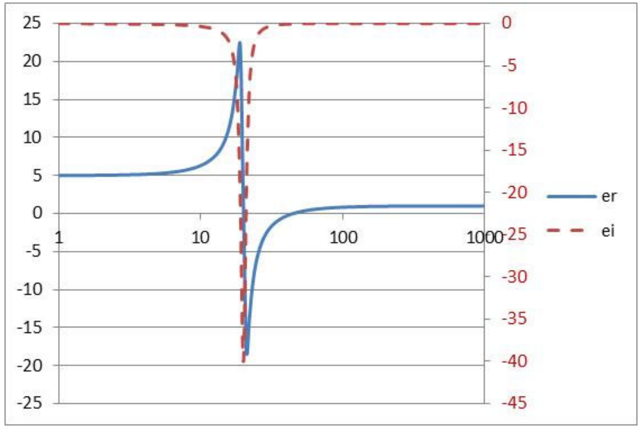
$$\omega_p = \sqrt{\frac{nq^2}{\varepsilon_0 m}}$$ **pulsazione di plasma**

Modello atomico: permettività elettrica relativa

$$\mathrm{Re}\left(\tilde{\varepsilon}_r\right) = 1 + \frac{\omega_p^{\,2}\left(\omega_0^{\,2} - \omega^2\right)}{\left(\omega_0^{\,2} - \omega^2\right)^2 + \left(2\omega_c\omega\right)^2}$$

$$\mathrm{Im}\left(\tilde{\varepsilon}_r\right) = -\omega_p^{\,2}\frac{2\omega_c\omega}{\left(\omega_0^{\,2} - \omega^2\right)^2 + \left(2\omega_c\omega\right)^2}$$

(ω_c = 1, ω_0 = 20, ω_p = 40 u.a.)



Interazione con la materia

$$\tilde{\varepsilon}_r = \varepsilon_r + i\varepsilon_i = 1 + \omega_p^2 \frac{\left(\omega_0^2 - \omega^2\right) - i\left(2\omega_c\omega\right)}{\left(\omega_0^2 - \omega^2\right)^2 + \left(2\omega_c\omega\right)^2}$$

$$\Downarrow$$

$$\tilde{n} = \sqrt{\tilde{\varepsilon}_r} = n_r + i n_i$$

$$\Downarrow$$

$$\tilde{k} = \frac{\omega}{c}\tilde{n} = \frac{\omega}{c}\sqrt{\tilde{\varepsilon}_r} = k_r + i k_i \qquad \begin{array}{l} k_i < 0 \\ k_r > 0 \end{array}$$

$$E = E_0 \cos\left(\omega t - kz\right)$$

$$\tilde{E} = E_0 e^{i\left(\omega t - \tilde{k}z\right)} = E_0 e^{i\left[\omega t - \left(k_r + i k_i\right)z\right]} = E_0 e^{k_i z} e^{i\left(\omega t - k_r z\right)}$$

dispersione cromatica

assorbimento

Interazione con la materia: materiali rarefatti

$$\tilde{\varepsilon}_r = \varepsilon_r + \mathrm{i}\varepsilon_i$$

$$\tilde{n} = \sqrt{\tilde{\varepsilon}_r} \approx \frac{1}{2} + \frac{1}{2}\tilde{\varepsilon}_r = 1 + \frac{\omega_p{}^2}{2}\frac{\left(\omega_0{}^2 - \omega^2\right) - \mathrm{i}\left(2\omega_c\omega\right)}{\left(\omega_0{}^2 - \omega^2\right)^2 + \left(2\omega_c\omega\right)^2}$$

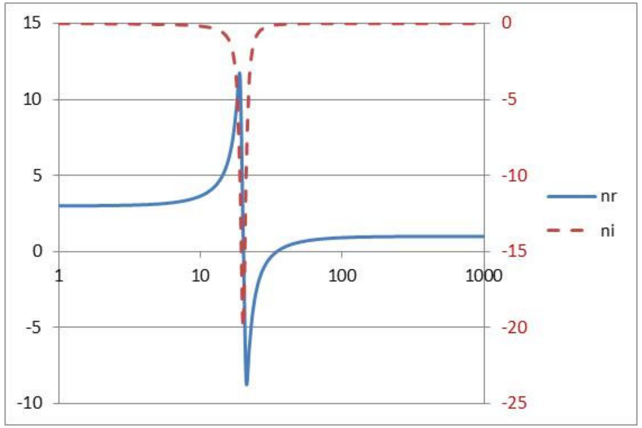
$$\downarrow$$

$$\text{se}\left|\tilde{\varepsilon}_r\right| \approx 1$$

$$n_r = 1 + \frac{\omega_p{}^2}{2}\frac{\left(\omega_0{}^2 - \omega^2\right)}{\left(\omega_0{}^2 - \omega^2\right)^2 + \left(2\omega_c\omega\right)^2}$$

$$n_i = -\frac{\omega_p{}^2}{2}\frac{2\omega_c\omega}{\left(\omega_0{}^2 - \omega^2\right)^2 + \left(2\omega_c\omega\right)^2}$$

($\omega_c = 1$, $\omega_0 = 20$, $\omega_p = 40$ u.a.)



Interazione con la materia: materiali rarefatti

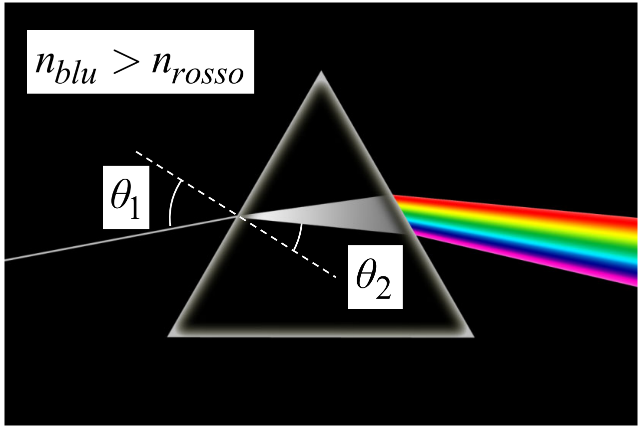
parte reale

$$E = E_0 e^{k_i z} e^{i(\omega t - k_r z)}$$

$$k_r = \frac{\omega}{c} n_r$$

$$n = 1 + \frac{\omega_p^{\,2}}{2} \frac{\left(\omega_0^{\,2} - \omega^2\right)}{\left(\omega_0^{\,2} - \omega^2\right)^2 + \left(2\omega_c \omega\right)^2}$$

indice di rifrazione (classico)



http://www.pinkfloyd.com

parte immaginaria

$$E = E_0 e^{k_i z} e^{i(\omega t - k_r z)}$$

$$\langle I_z \rangle \propto E^2 = E_0^{\,2} e^{2 k_i z} = E_0^{\,2} e^{2\frac{\omega}{c} n_i z}$$

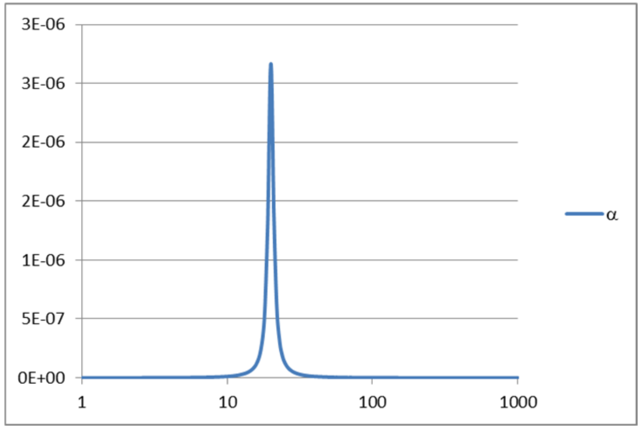
$$\alpha = \frac{\omega_p^{\,2}}{c} \frac{2\omega_c \omega^2}{\left(\omega_0^{\,2} - \omega^2\right)^2 + \left(2\omega_c\omega\right)^2}$$

coefficiente di assorbimento

legge di Lambert-Beer

$$\langle I_z \rangle = \langle I_0 \rangle e^{-\alpha z}$$

(ω_c = 1, ω_0 = 20, ω_p = 40 u.a.)



Interazione con la materia: materiali dispersivi

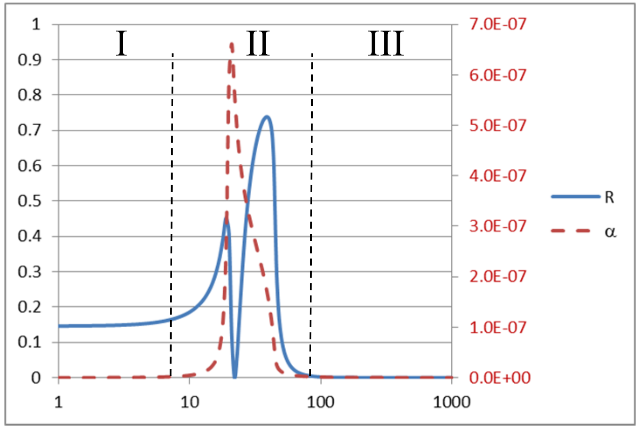
$$\tilde{\varepsilon}_r = 1 + \omega_p^2 \frac{\left(\omega_0^2 - \omega^2\right) - \mathrm{i}\left(2\omega_c\omega\right)}{\left(\omega_0^2 - \omega^2\right)^2 + \left(2\omega_c\omega\right)^2}$$

$$\tilde{n} = \sqrt{\tilde{\varepsilon}_r} = n_r + \mathrm{i}n_i \quad \begin{cases} n_r = \mathrm{Re}\left(\tilde{n}\right) = \mathrm{Re}\left(\sqrt{\tilde{\varepsilon}_r}\right) \\ n_i = \mathrm{Im}\left(\tilde{n}\right) = \mathrm{Im}\left(\sqrt{\tilde{\varepsilon}_r}\right) \end{cases}$$

$$\alpha = -2\frac{\omega}{c} n_i$$ **coeff. di assorbimento**

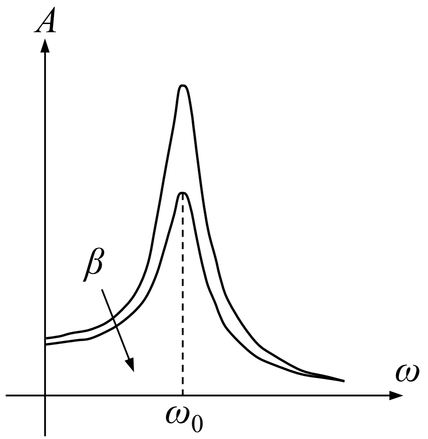
$$R = \left(\frac{n_1 - n_r}{n_1 + n_r}\right)^2$$ **riflettanza**

(n_1 = 1; ω_c = 1, ω_0 = 20, ω_p = 40 u.a.)



Interazione con la materia: materiali diffondenti

$$A = \frac{qE_0}{m} \frac{1}{\sqrt{\left(\omega_0{}^2 - \omega^2\right)^2 + \left(2\omega_c\omega\right)^2}}$$



$$\langle P_{rad} \rangle = \frac{1}{12\pi\varepsilon_0} \frac{p^2\omega^4}{c^3} = \frac{1}{12\pi\varepsilon_0} \frac{q^2 A^2 \omega^4}{c^3} = \frac{1}{12\pi\varepsilon_0} \frac{q^2\omega^4}{c^3} \left(\frac{qE_0}{m}\right)^2 \frac{1}{\left(\omega_0{}^2 - \omega^2\right)^2 + \left(2\omega_c\omega\right)^2}$$

$$\langle I \rangle = \frac{1}{2} c\varepsilon_0 E_0{}^2 \qquad \langle P_{rad} \rangle = \langle I \rangle \sigma$$

sezione d'urto di diffusione

Interazione con la materia: materiali diffondenti

$$\sigma_{diff} = \frac{8\pi r_0^2}{3} \frac{\omega^4}{\left(\omega_0^2 - \omega^2\right)^2 + \left(2\omega_c \omega\right)^2}$$

$$\left(\omega_c <<\right) \omega >> \omega_0$$

$$r_0 = \left(\frac{q^2}{4\pi\varepsilon_0}\right)\frac{1}{mc^2} = 2.82 \cdot 10^{-15} \text{ m}$$

raggio classico dell'elettrone

$$\sigma_{diff} \approx \frac{8\pi r_0^2}{3} = \sigma_T$$

legge di Thomson

Interazione con la materia: materiali diffondenti

$$\sigma_{diff} = \frac{8\pi r_0^2}{3} \frac{\omega^4}{\left(\omega_0^2 - \omega^2\right)^2 + \left(2\omega_c\omega\right)^2}$$

$$\left(\omega_c <<\right) \omega << \omega_0$$

$$\sigma_{diff} \approx \sigma_T \frac{\omega^4}{\omega_0^4}$$

legge di Rayleigh