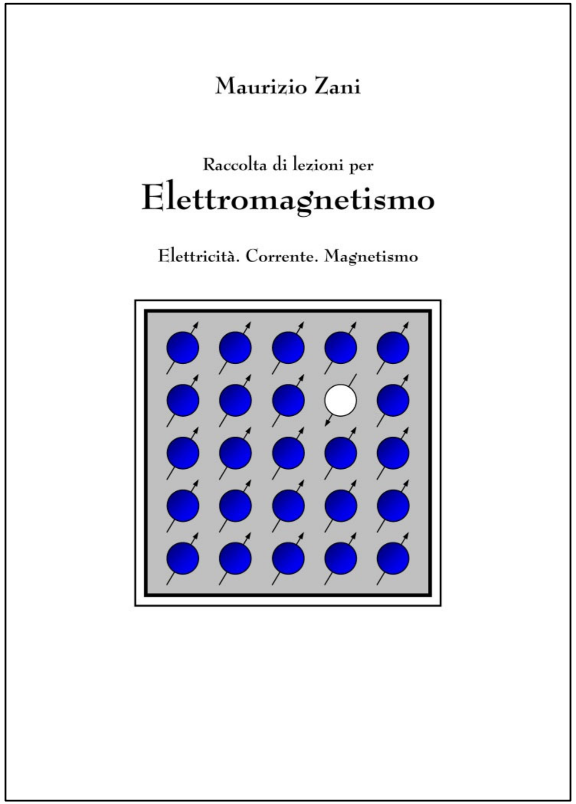
POLITECNICO
MILANO 1863

Elettromagnetismo

Elettricità. Corrente. Magnetismo

Maurizio Zani

Elettromagnetismo

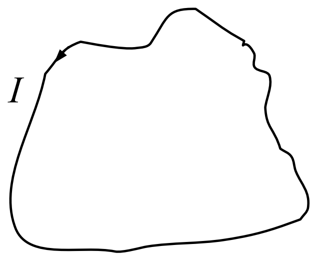


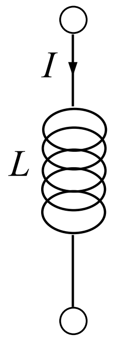
http://www.maurizIozani.it/wp/?p=1128

Elettromagnetismo

Induttanza

induttore





rapporto causa/effetto

dipende da materiale/geometria

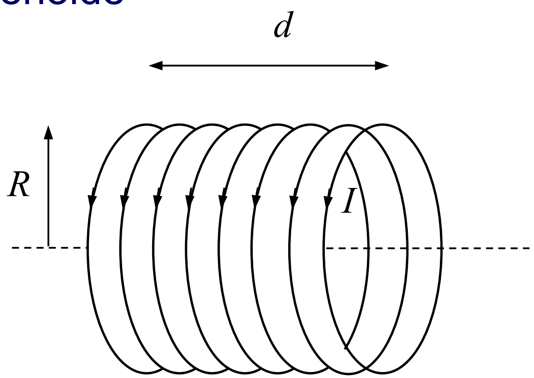
$$L = \frac{\Phi\left(\vec{B}\right)}{I}$$

induttanza

$$\left[L\right] = \frac{\left[\Phi\left(\vec{B}\right)\right]}{\left[I\right]} = \frac{\mathrm{Wb}}{\mathrm{A}} = \mathrm{H}$$

henry

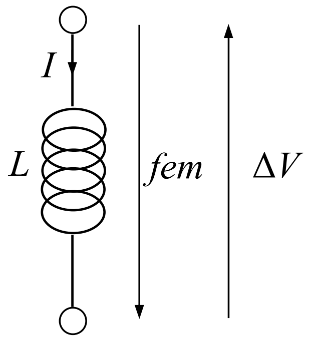
es. solenoide



$$B = \mu_0 n I = \mu_0 \frac{N}{d} I$$

$$L = \frac{\Phi\left(\vec{B}\right)}{I} = \mu_0 \frac{N^2 S}{d}$$

Induttanza



condizioni variabili nel tempo

$$fem = -\frac{\mathrm{d}\Phi\left(\vec{B}\right)}{\mathrm{d}t} = -\frac{\mathrm{d}\left(LI\right)}{\mathrm{d}t} = -L\frac{\mathrm{d}I}{\mathrm{d}t}$$

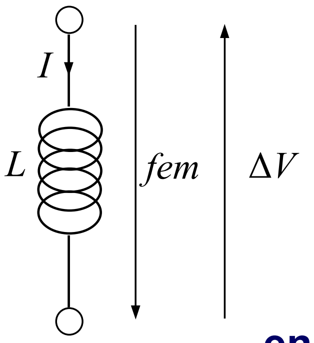
$$\Delta V = -fem = L\frac{\mathrm{d}I}{\mathrm{d}t}$$

energia magnetica

$$P = fem \cdot I = -L\frac{\mathrm{d}I}{\mathrm{d}t} \cdot I = -L\frac{1}{2}\frac{\mathrm{d}I^2}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}LI^2\right)$$

$$E_m = \frac{1}{2}LI^2$$

Energia magnetica



energia legata
alla corrente

energia magnetica

$$E_m = \frac{1}{2} L I^2$$

energia legata
al campo magnetico

$$E_m = \frac{1}{2} L I^2 = \frac{1}{2} \mu_0 \frac{N^2 S}{\mathrm{d}} \cdot \left(\frac{Bd}{\mu_0 N} \right)^2 = \frac{1}{2} \frac{1}{\mu_0} B^2 \cdot Sd$$

volume

solenoide

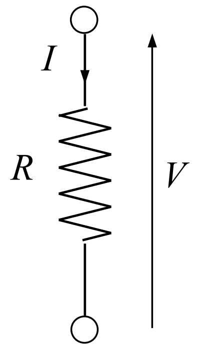
$$L = \mu_0 \frac{N^2 S}{d}$$

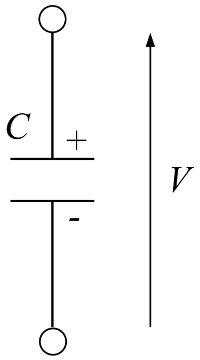
$$B = \mu_0 n I = \mu_0 \frac{N}{d} I$$

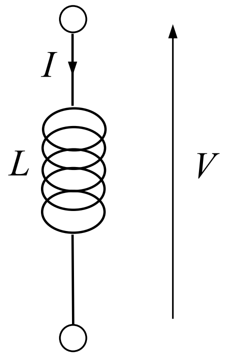
$$\rho_m = \frac{E_m}{V} = \frac{1}{2} \frac{1}{\mu_0} B^2$$

densità di energia magnetica

Bipoli







$$R = \frac{V}{I} = \rho \frac{L}{S}$$

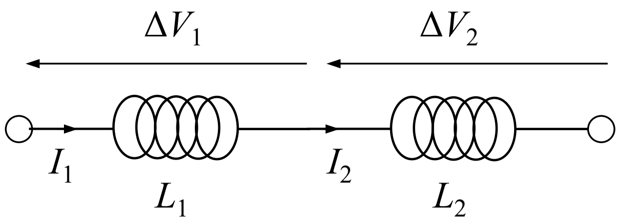
$$C = \frac{q}{V} = \varepsilon_0 \frac{S}{d}$$

$$L = \frac{\Phi\left(\vec{B}\right)}{I} = \mu_0 \frac{S}{d}$$

$$\frac{\mathrm{d}q}{\mathrm{d}t} = I = C\frac{\mathrm{d}V}{\mathrm{d}t}$$

$$\frac{\mathrm{d}\Phi}{\mathrm{d}t} = V = L\frac{\mathrm{d}I}{\mathrm{d}t}$$

Serie e parallelo

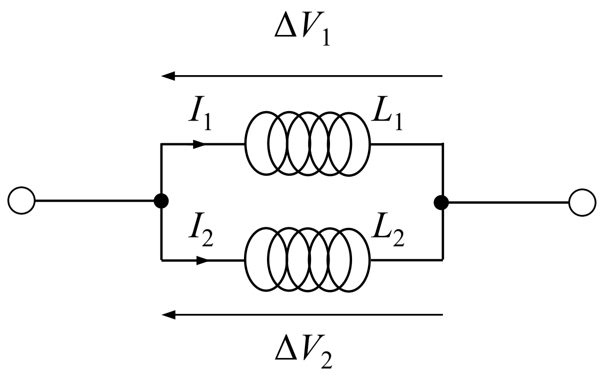


induttori in serie

$$I_1 = I_2 = I$$

$$\Delta V = \Delta V_1 + \Delta V_2 = L_1 \frac{dI_1}{dt} + L_2 \frac{dI_2}{dt} = (L_1 + L_2)\frac{dI}{dt}$$

$$L_{eq} = \frac{\Delta V}{dI/dt} = \frac{(L_1 + L_2)\, dI/dt}{dI/dt} = L_1 + L_2$$



induttori in parallelo

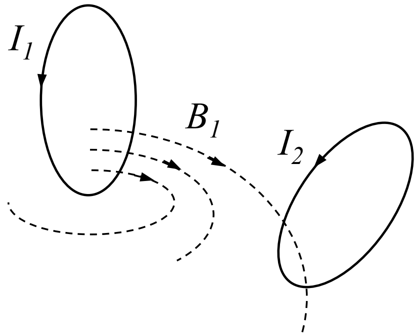
$$\Delta V_1 = \Delta V_2 = \Delta V$$

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} = \frac{\Delta V_1}{L_1} + \frac{\Delta V_2}{L_2} = \Delta V\left(\frac{1}{L_1} + \frac{1}{L_2}\right)$$

$$\frac{1}{L_{eq}} = \frac{dI/dt}{\Delta V} = \frac{\Delta V\left(1/L_1 + 1/L_2\right)}{\Delta V} = \frac{1}{L_1} + \frac{1}{L_2}$$

Mutua induttanza

M_{ij} dipendono da materiale/geometria

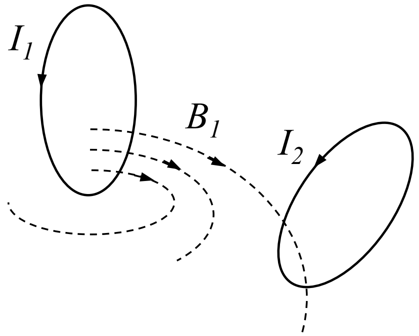


$$\begin{cases} \Phi_1 = M_{11}I_1 + M_{12}I_2 \\ \Phi_2 = M_{21}I_1 + M_{22}I_2 \end{cases}$$

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \qquad \begin{cases} M_{ii} = L_i > 0 \\ M_{ij} = M_{ji} \\ M_{ij} \le L_i L_j \end{cases}$$

coefficienti di induttanza

Mutua induttanza



$$\begin{cases} \Phi_1 = L_1 I_1 + M_{12} I_2 \\ \Phi_2 = M_{21} I_1 + L_2 I_2 \end{cases}$$

$$M_{21} = M_{12}$$

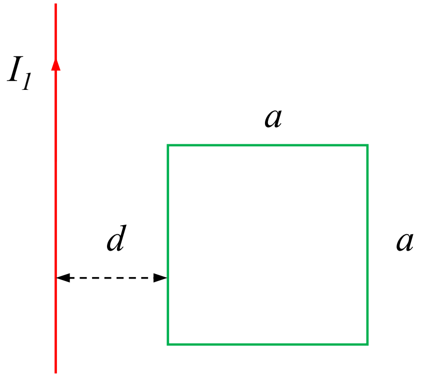
$$\Phi_2\left(\vec{B}\right) = \int \vec{B}_1 \cdot \mathrm{d}\vec{S}_2 = \int \mathrm{rot}\left(\vec{A}_1\right) \cdot \mathrm{d}\vec{S}_2 = \oint \vec{A}_1 \cdot \mathrm{d}\vec{r}_2 = \oint \left(\frac{\mu_0}{4\pi} \int \frac{\vec{J}_1}{r} \mathrm{d}V \right) \cdot \mathrm{d}\vec{r}_2 =$$

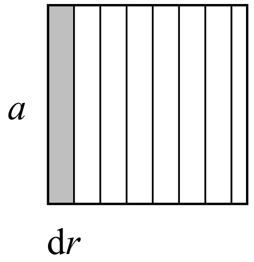
$$= \frac{\mu_0}{4\pi} \oint \left(\oint \frac{I_1}{r} \mathrm{d}\vec{r}_1 \right) \cdot \mathrm{d}\vec{r}_2 = \frac{\mu_0}{4\pi} I_1 \oint \left(\oint \frac{\mathrm{d}\vec{r}_1 \cdot \mathrm{d}\vec{r}_2}{r} \right)$$

$$M_{21} = \frac{\Phi_2\left(\vec{B}\right)}{I_1} = \frac{\mu_0}{4\pi} \oint \left(\oint \frac{\mathrm{d}\vec{r}_1 \cdot \mathrm{d}\vec{r}_2}{r} \right) = M_{12}$$

Mutua induttanza

es. filo rettilineo infinito e spira





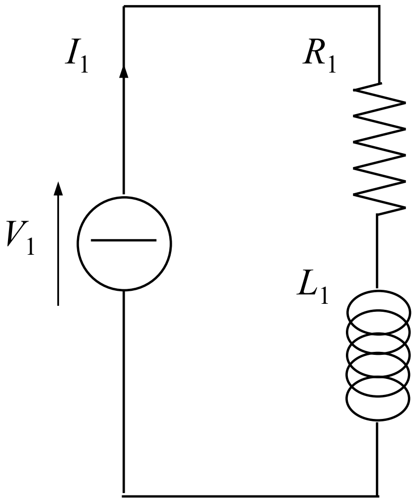
$$B_1 = \frac{\mu_0 I_1}{2\pi r}$$

$$\mathrm{d}S = a\,\mathrm{d}r$$

$$\Phi_2\left(\vec{B}\right) = \int \vec{B}_1 \cdot \mathrm{d}\vec{S} = \int B_1\,\mathrm{d}S = \frac{\mu_0 I_1}{2\pi}\int_d^{d+a} \frac{1}{r}\,a\,\mathrm{d}r = \frac{\mu_0 I_1 a}{2\pi}\ln\left(\frac{d+a}{d}\right)$$

$$M_{12} = \frac{\Phi_2\left(\vec{B}\right)}{I_1} = \frac{\mu_0 a}{2\pi}\ln\left(\frac{d+a}{d}\right)$$

Mutua induttanza



$$V_1 = R_1 I_1 + L_1 \frac{\mathrm{d}I_1}{\mathrm{d}t}$$

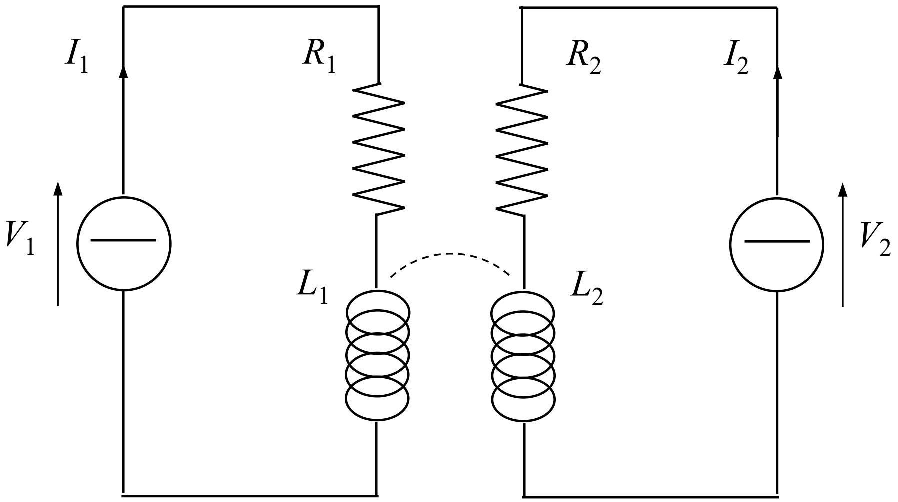
$$\Big\downarrow \cdot I_1 \mathrm{d}t$$

$$\left(V_1 I_1\right)\mathrm{d}t = \left(R_1 I_1^{\,2}\right)\mathrm{d}t + L_1 I_1 \mathrm{d}I_1$$

$$\mathrm{d}E_{gen} \qquad \mathrm{d}E_{diss} \qquad \mathrm{d}\left(\frac{1}{2} L_1 I_1^{\,2}\right)$$

$$\mathrm{d}E_{magn}$$

Mutua induttanza

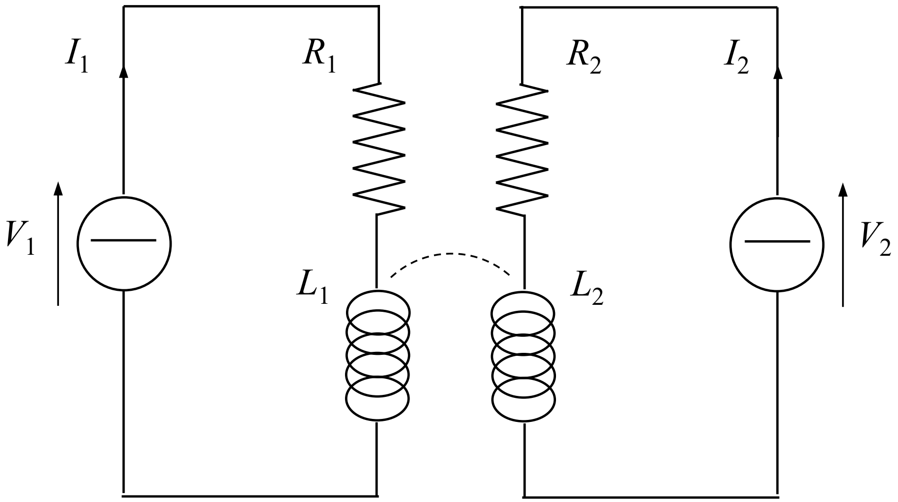


$$\begin{cases} V_1 = R_1 I_1 + L_1 \dfrac{dI_1}{dt} + M \dfrac{dI_2}{dt} & \xrightarrow{} \cdot I_1 dt \\ V_2 = R_2 I_2 + L_2 \dfrac{dI_2}{dt} + M \dfrac{dI_1}{dt} & \xrightarrow{} \cdot I_2 dt \end{cases}$$

$$\begin{cases} V_1 I_1 dt = R_1 I_1^2 dt + L_1 I_1 dI_1 + M I_1 dI_2 \\ V_2 I_2 dt = R_2 I_2^2 dt + L_2 I_2 dI_2 + M I_2 dI_1 \end{cases}$$

$$\left(V_1 I_1 + V_2 I_2\right) dt = \left(R_1 I_1^2 + R_2 I_2^2\right) dt + L_1 I_1 dI_1 + L_2 I_2 dI_2 + M I_1 dI_2 + M I_2 dI_1$$

Mutua induttanza



$$\underbrace{(V_1 I_1 + V_2 I_2)\,\mathrm{d}t}_{\mathrm{d}E_{gen}} = \underbrace{(R_1 I_1^2 + R_2 I_2^2)\,\mathrm{d}t}_{\mathrm{d}E_{diss}} + \overbrace{\underbrace{L_1 I_1 \mathrm{d}I_1 + L_2 I_2 \mathrm{d}I_2}_{\substack{\mathrm{d}\left(\frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2\right) \\ \mathrm{d}E_{magn\ propria}}} + \underbrace{M I_1 \mathrm{d}I_2 + M I_2 \mathrm{d}I_1}_{\substack{\mathrm{d}(M I_1 I_2) \\ \mathrm{d}E_{magn\ mutua}}}}^{\mathrm{d}E_{magn}}$$