POLITECNICO
MILANO 1863

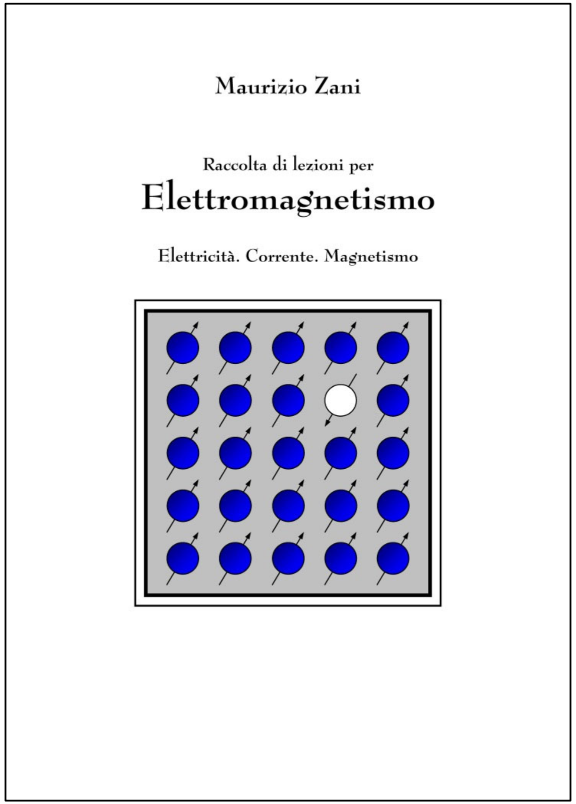
Elettromagnetismo

Elettricità. Corrente. Magnetismo

Maurizio Zani

Sommario

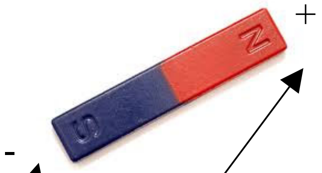
Elettromagnetismo



http://www.mauriziozani.it/wp/?p=1128

Elettromagnetismo

Magnetizzazione



il materiale si magnetizza?

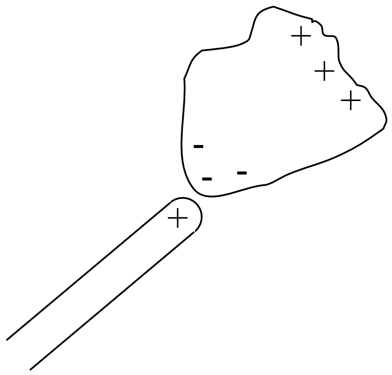
- no: **materiale "amagnetico"**
  - come rame e alluminio
- sì: **materiale magnetico**
  - come il ferro

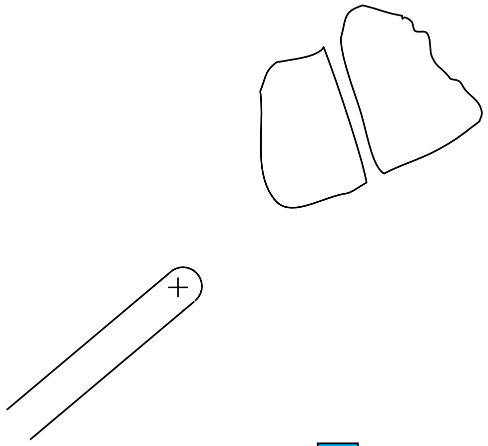
magnete permanente

- polo (p)
  - due tipi (S e N), vale algebra
- interazione
  - tipo diverso: attrazione
  - stesso tipo: repulsione

Magnetizzazione

materiali magnetici





prossimità (senza contatto)

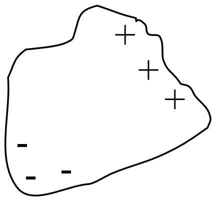
magnetizzazione
localizzata e temporanea

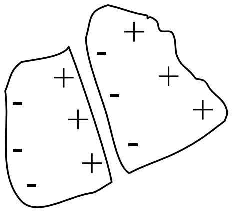
prossimità e taglio

nessuna
magnetizzazione

Magnetizzazione

magneti permanenti



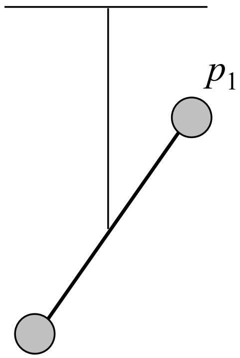


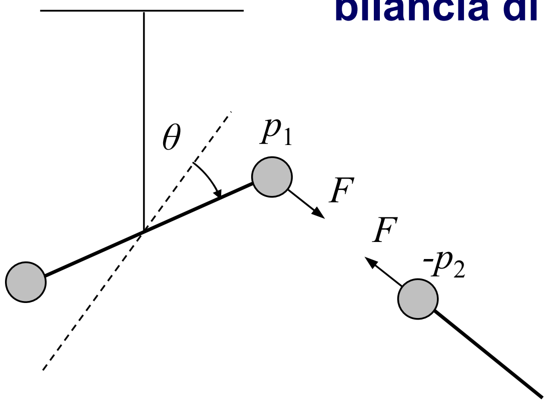
taglio

magnetizzazione
permanente

magnetizzazione permanente
in ogni frazione

Magnetizzazione: forza magnetica





bilancia di torsione

costante magnetica

$$\vec{F}_m = k_m \frac{p_1 p_2}{r^2} \vec{u}_r$$

forza magnetica (di Lorentz)

forza fondamentale

$$k_m = \frac{\mu_0}{4\pi} = 10^{-7} \text{ N / A}^2$$

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ N / A}^2$$

valore esatto

permeabilità magnetica

Magnetizzazione: forza magnetica

$$c = 299\ 792\ 458\ \mathrm{m/s}$$

$$\sqrt{\frac{k_e}{k_m}} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c$$

velocità della luce

$$k_m = \frac{\mu_0}{4\pi}$$

$$k_e = \frac{1}{4\pi\varepsilon_0}$$

$$\vec{F}_m = k_m \frac{p_1 p_2}{r^2} \vec{u}_r$$

forza magnetica (di Lorentz)

$$\vec{F}_e = k_e \frac{q_1 q_2}{r^2} \vec{u}_r$$

forza elettrica (di Coulomb)

Magnetizzazione: campo magnetico

$$\vec{F}_m = \frac{\mu_0}{4\pi}\frac{p_1 p_2}{r^2}\vec{u}_r$$

$$[B] = \frac{[F]}{[p]} = \frac{\text{N}}{\text{Am}} = \text{T}$$

tesla

oggetto

$$\underbrace{\vec{F}_m}_{\text{effetto}} = \frac{\mu_0}{4\pi}\frac{p_1 p_2}{r^2}\vec{u}_r = p_1\left(\frac{\mu_0}{4\pi}\frac{p_2}{r^2}\vec{u}_r\right) = p_1\vec{B}_2$$

effetto

causa

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{p_2}{r^2}\vec{u}_r$$

campo magnetico

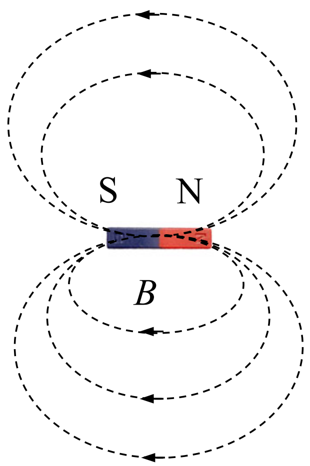
$$\vec{F}_1 = \sum \vec{F}_i = \sum p_1\vec{B} = p_1\sum \vec{B}_i = p_1\vec{B}$$

$$\left\{\begin{array}{l} \vec{B} = \sum \vec{B}_i \\ \\ \vec{B} = \int \mathrm{d}\vec{B} \end{array}\right.$$

vale la sovrapposizione degli effetti

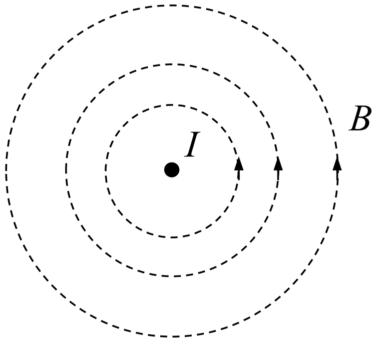
Magnetizzazione: campo magnetico

magnete



orientazione della
limatura di ferro

corrente



linee di flusso

- linee orientate, tangenti (direzione) e concordi (verso) al campo
- si addensano dove il campo è più intenso
- non si incrociano mai
- partono (sorgente) e terminano (pozzo) sui poli o all'infinito

Forza magnetica

$$\vec{F}_m = q\vec{v} \times \vec{B}$$

forza magnetica (di Lorentz)

$$[B] = \frac{[F]}{[q][v]} = \frac{\mathrm{N}}{\mathrm{C}\,\mathrm{m}/\mathrm{s}} = \frac{\mathrm{N}}{\mathrm{Am}} = \mathrm{T}$$

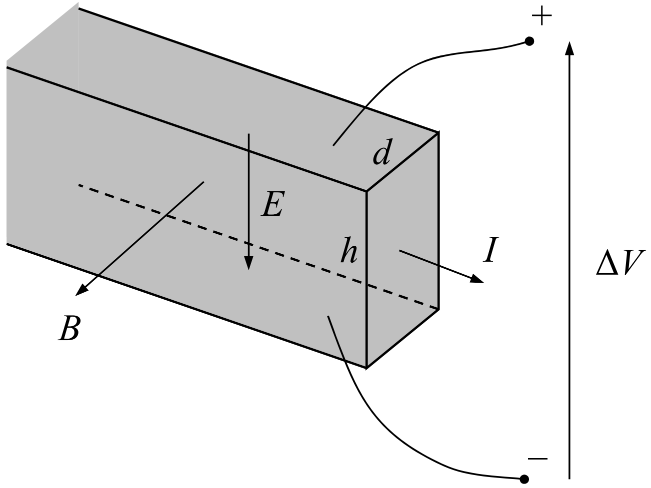
tesla

$$W = \int \vec{F}_m \cdot \mathrm{d}\vec{r} = \Delta K = 0$$

$$K = \frac{1}{2}mv^2 = \text{costante}$$

la forza magnetica non compie lavoro

Forza magnetica: effetto Hall



$$F_m = F_e \qquad qvB = qE = q\frac{\Delta V}{h}$$

$$\Delta V = vBh$$

il segno dipende dal segno dei portatori

$$J = nqv = \frac{I}{hd} \qquad \Delta V = vBh = \frac{I}{dnq}B$$

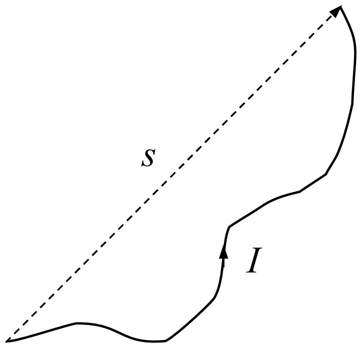
$$\frac{\Delta V}{I} = \frac{B}{dnq}$$

resistenza Hall

Forza magnetica: II legge elementare di Laplace

$$\mathrm{d}\vec{F} = \mathrm{d}q\,\vec{v}\times\vec{B} = \mathrm{d}q\frac{\mathrm{d}\vec{s}}{\mathrm{d}t}\times\vec{B} = \frac{\mathrm{d}q}{\mathrm{d}t}\mathrm{d}\vec{s}\times\vec{B} = I\mathrm{d}\vec{s}\times\vec{B}$$

II legge elementare di Laplace



$$\vec{F} = \int \mathrm{d}\vec{F} = I\int \mathrm{d}\vec{s}\times\vec{B}$$

$$\vec{F} = I\int \mathrm{d}\vec{s}\times\vec{B} = I\left(\int \mathrm{d}\vec{s}\right)\times\vec{B} = I\,\vec{s}\times\vec{B}$$

campo uniforme

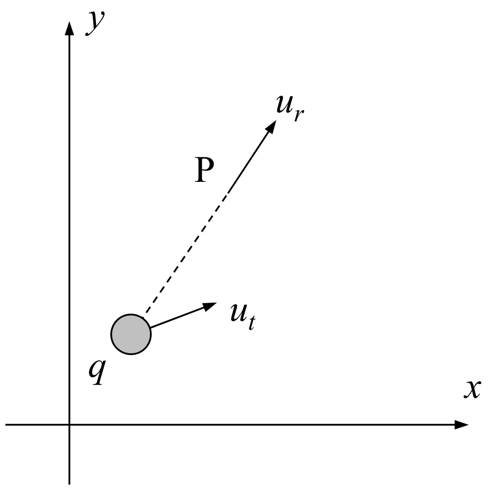
$$\vec{F} = I\int \mathrm{d}\vec{s}\times\vec{B} = I\left(\oint \mathrm{d}\vec{s}\right)\times\vec{B} = 0$$

circuito chiuso

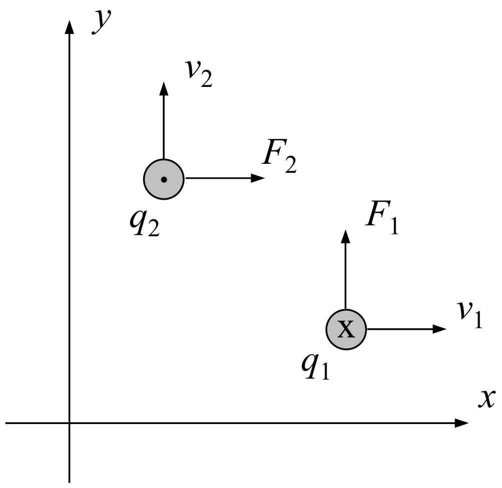
Campo magnetico

$$\vec{B} = \frac{\mu_0}{4\pi} q v \frac{\vec{u}_t \times \vec{u}_r}{r^2} \qquad [B] = \mathrm{T}$$

campo magnetico **tesla**



$$\vec{F}_m = q\vec{v} \times \vec{B}$$

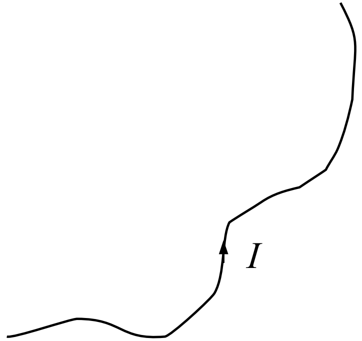


non vale il III principio della dinamica

Campo magnetico: I legge elementare di Laplace

$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}\mathrm{d}q\, v\frac{\vec{u}_t\times\vec{u}_r}{r^2} = \frac{\mu_0}{4\pi}\mathrm{d}q\frac{\mathrm{d}s}{\mathrm{d}t}\frac{\vec{u}_t\times\vec{u}_r}{r^2} =$$

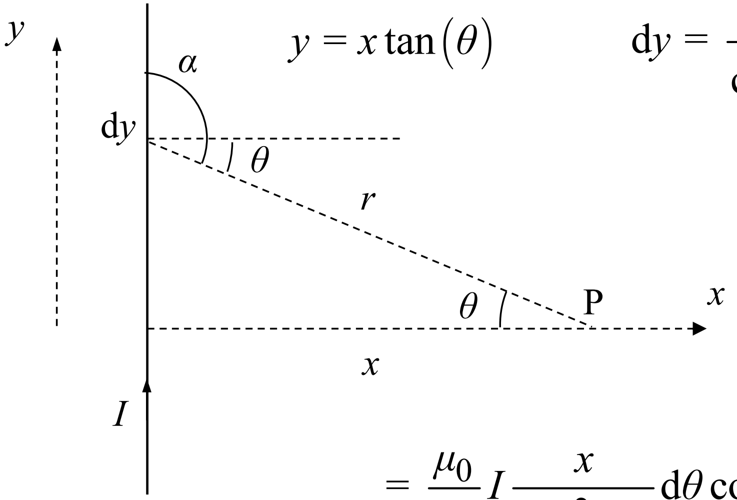
$$= \frac{\mu_0}{4\pi}\frac{\mathrm{d}q}{\mathrm{d}t}\mathrm{d}s\frac{\vec{u}_t\times\vec{u}_r}{r^2} = \frac{\mu_0}{4\pi}I\,\mathrm{d}s\frac{\vec{u}_t\times\vec{u}_r}{r^2}$$



I legge elementare di Laplace

$$\vec{B} = \int \mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}I\int \mathrm{d}s\frac{\vec{u}_t\times\vec{u}_r}{r^2}$$

filo rettilineo infinito



$$y = x\tan(\theta) \qquad dy = \frac{x}{\cos^2(\theta)} d\theta \qquad \sin(\alpha) = \sin\left(\frac{\pi}{2} + \theta\right) = \cos(\theta)$$

$$dB = \frac{\mu_0}{4\pi} I\, dy \frac{\sin(\alpha)}{r^2} =$$

$$= \frac{\mu_0}{4\pi} I \frac{x}{\cos^2(\theta)} d\theta \cos(\theta) \frac{\cos^2(\theta)}{x^2} = \frac{\mu_0}{4\pi}\frac{I}{x}\cos(\theta)\, d\theta$$

$$r = \frac{x}{\cos(\theta)}$$

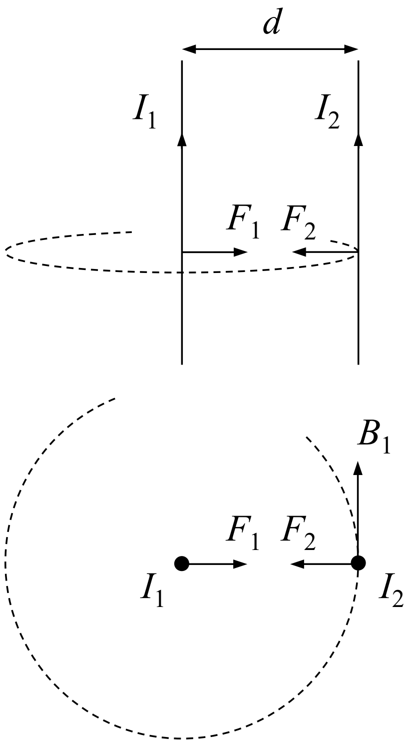
$$B = \int dB = \frac{\mu_0}{4\pi}\frac{I}{x}\int_{-\pi/2}^{+\pi/2} \cos(\theta)\, d\theta = \frac{\mu_0}{4\pi}\frac{I}{x}\left[\sin(\theta)\right]_{-\pi/2}^{+\pi/2} = \frac{\mu_0}{2\pi}\frac{I}{x}$$

legge di Biot-Savart

Campo magnetico: forza tra correnti

$$B_1 = \frac{\mu_0}{2\pi d} I_1$$

$$F_2 = I_2 L B_1 = I_2 L \frac{\mu_0}{2\pi d} I_1 = L \frac{\mu_0}{2\pi d} I_1 I_2$$



$$\frac{F_2}{L} = \frac{\mu_0}{2\pi d} I_1 I_2$$

vale il III principio della dinamica

Teorema di Gauss

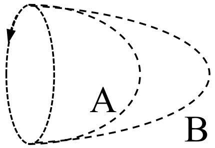
$$\Phi\left(\vec{B}\right) = \oint \vec{B}\cdot \mathrm{d}\vec{S} = 0$$

teorema di Gauss

le linee del campo sono chiuse

non vi sono poli magnetici isolati

$$\Phi\left(\vec{B}\right) = \int_{A} \vec{B}\cdot \mathrm{d}\vec{S} = \int_{B} \vec{B}\cdot \mathrm{d}\vec{S}$$



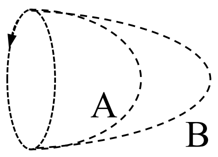
$$\left[\Phi\left(\vec{B}\right)\right] = [B][S] = \mathrm{Tm}^2 = \mathrm{Wb}$$

weber

Teorema di Gauss

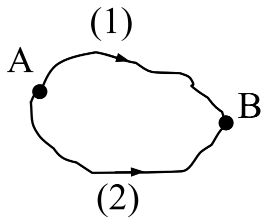
$$\Phi\left(\vec{B}\right) = \oint \vec{B} \cdot \mathrm{d}\vec{S} = 0$$

$$\Phi\left(\vec{B}\right) = \int_{\mathrm{A}} \vec{B} \cdot \mathrm{d}\vec{S} = \int_{\mathrm{B}} \vec{B} \cdot \mathrm{d}\vec{S}$$



$$\Lambda\left(\vec{E}\right) = \oint \vec{E} \cdot \mathrm{d}\vec{r} = 0$$

$$\Lambda\left(\vec{E}\right) = \underset{(1)}{\int_{\mathrm{A}}^{\mathrm{B}}} \vec{E} \cdot \mathrm{d}\vec{r} = \underset{(2)}{\int_{\mathrm{A}}^{\mathrm{B}}} \vec{E} \cdot \mathrm{d}\vec{r}$$

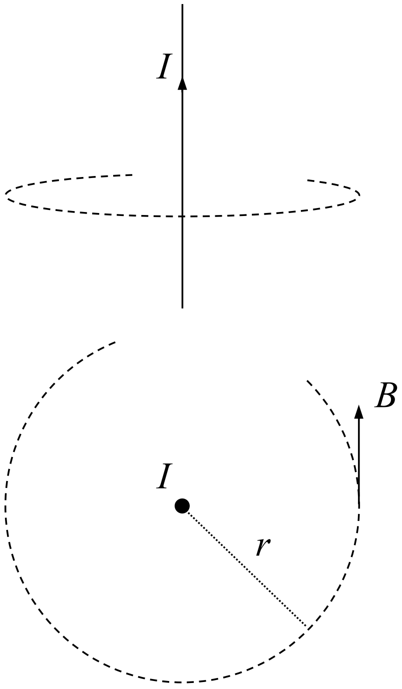


Teorema di Ampère: linea circolare

$$L = 2\pi r$$

$$\Lambda\left(\vec{B}\right) = \oint \vec{B}\cdot \mathrm{d}\vec{r} = \oint B\,\mathrm{d}r = B\oint \mathrm{d}r = BL$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\vec{u}_\theta$$



$$\Lambda\left(\vec{B}\right) = \oint \vec{B}\cdot \mathrm{d}\vec{r} = \mu_0 I$$

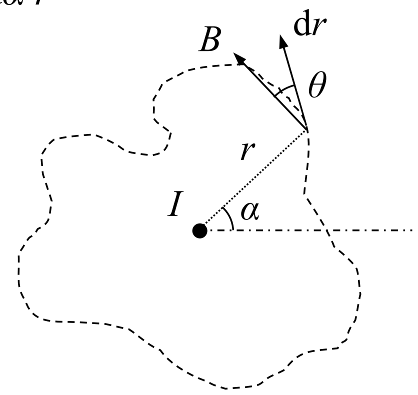
teorema di Ampère

Teorema di Ampère: linea generica

$$\mathrm{d}r\cos(\theta) = \mathrm{d}r' = \mathrm{d}\alpha\, r$$

$$\Lambda(\vec{B}) = \oint \vec{B}\cdot \mathrm{d}\vec{r} = \oint B\,\mathrm{d}r\cos(\theta)$$

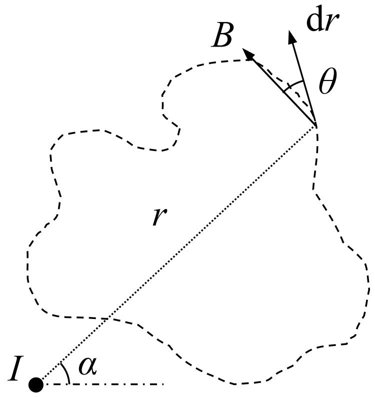
$$\vec{B} = \frac{\mu_0 I}{2\pi r}\vec{u}_\theta$$



$$\Lambda(\vec{B}) = \oint \mathrm{d}\Lambda = \frac{\mu_0 I}{2\pi}\oint \mathrm{d}\alpha$$

corrente interna $\oint \mathrm{d}\alpha = 2\pi$

corrente esterna $\oint \mathrm{d}\alpha = 0$



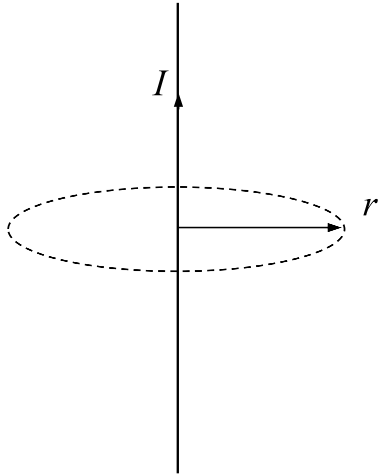
Teorema di Ampère

"La circuitazione del campo magnetico creato da più correnti dipende unicamente dalla corrente concatenata dalla linea scelta, e ne risulta proporzionale secondo un fattore μ_0"

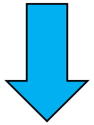
$$\Lambda\left(\vec{B}\right) = \oint \vec{B} \cdot \mathrm{d}\vec{r} = \mu_0 I_c$$

sempre valido, non sempre utile

filo rettilineo infinito



per simmetria, il campo magnetico è
- ortogonale rispetto al filo
- invariante per traslazione lungo il filo
- invariante per rotazione attorno al filo



simmetria "cilindrica"

$$\Lambda\left(\vec{B}\right) = \underbrace{\oint \vec{B}\cdot \mathrm{d}\vec{r}}_{\text{circuitaz.}} = \underbrace{\mu_0 I_c}_{\text{Ampère}}$$

$$\Lambda\left(\vec{B}\right) = \oint \vec{B}\cdot \mathrm{d}\vec{r} = \oint B\,\mathrm{d}r = B\oint \mathrm{d}r = B\,2\pi r$$

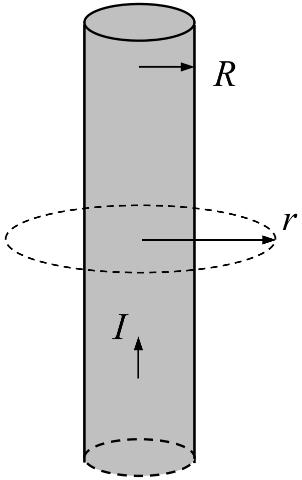
$$= \mu_0 I_c = \mu_0 I$$

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

legge di Biot-Savart

Teorema di Ampère

cilindro rettilineo infinito



$$\Lambda\left(\vec{B}\right) = \oint \vec{B}\cdot \mathrm{d}\vec{r} = B\,2\pi r = \mu_0 I_c$$

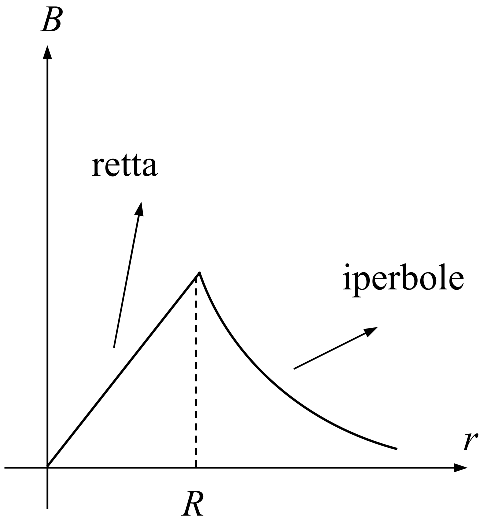
$$r > R: \quad I_c = I \qquad B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

$$r < R: \quad I_c = JS = \frac{\mathrm{I}}{\pi R^2}\pi r^2 \qquad B = \frac{\mu_0 I}{2\pi R^2} r$$

per simmetria, il campo magnetico è

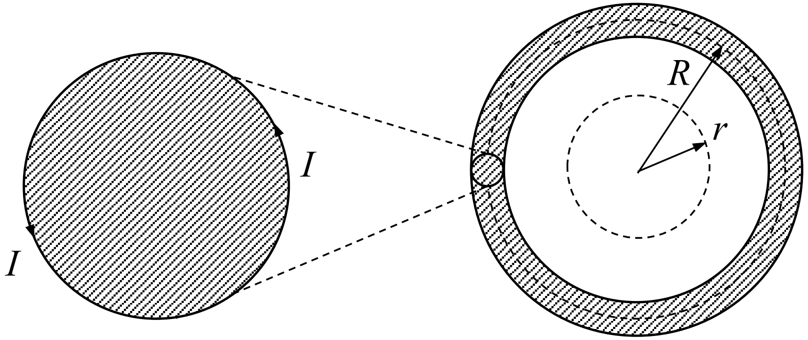
- ortogonale rispetto al filo
- invariante per traslazione lungo il filo
- invariante per rotazione attorno al filo

simmetria "cilindrica"



Teorema di Ampère

solenoide toroidale



$$\Lambda\left(\vec{B}\right) = \oint \vec{B}\cdot \mathrm{d}\vec{r} = B\ 2\pi r = \mu_0 I_c$$

$r \gg R$: $\quad I_c = NI - NI = 0 \qquad B = 0$

$r \ll R$: $\quad I_c = 0 \qquad B = 0$

$r \approx R$: $\quad I_c = NI \qquad B = \dfrac{\mu_0 NI}{2\pi r} = \mu_0 n I$

per simmetria, il campo magnetico è

- invariante per rotazione

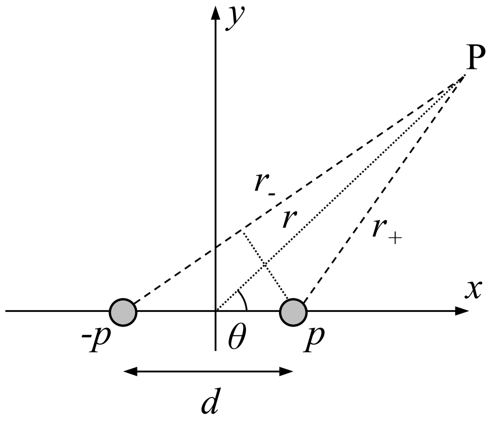
simmetria rotazionale

$$n = \frac{N}{2\pi r}$$

densità di spire

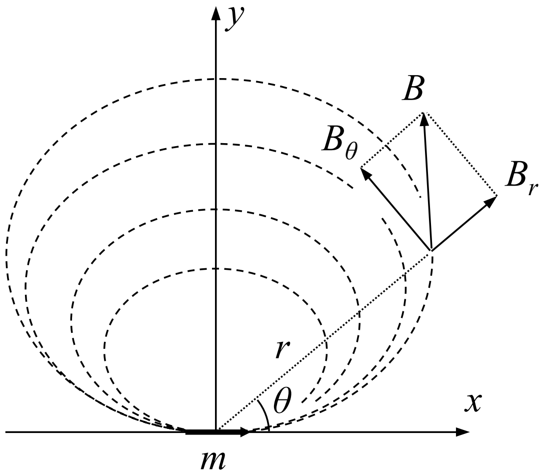
Dipolo magnetico: interazioni create

$$\begin{cases} B_r = \dfrac{\mu_0}{4\pi}\dfrac{2m\cos(\theta)}{r^3} \\ B_\theta = \dfrac{\mu_0}{4\pi}\dfrac{m\sin(\theta)}{r^3} \end{cases}$$



$$\vec{m} = p\vec{d} \qquad [m] = [p][d] = \mathrm{Am}^2$$

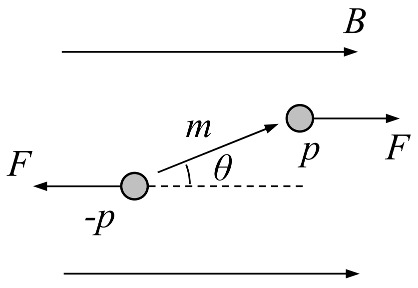
momento di dipolo magnetico



Dipolo magnetico: interazioni subite

$$\vec{M} = \vec{d} \times \vec{F} = \vec{d} \times p\vec{B} = p\vec{d} \times \vec{B} = \vec{m} \times \vec{B}$$

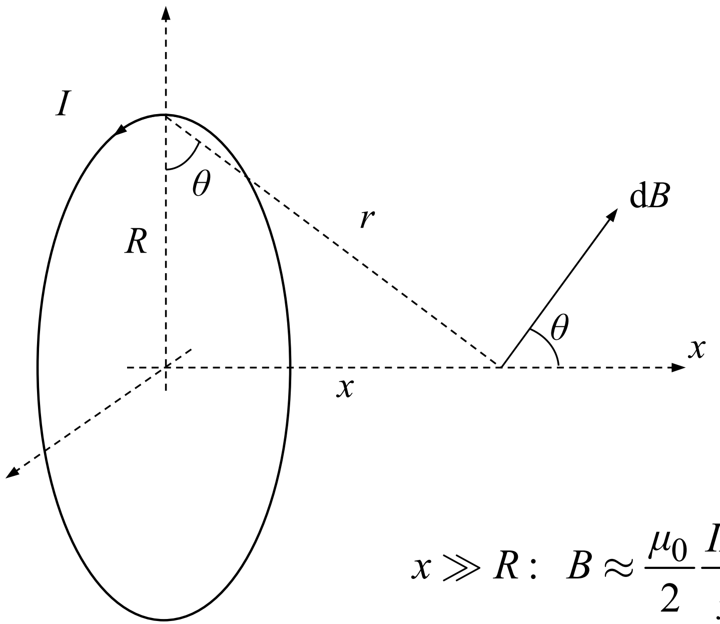
$$U = -\vec{m} \cdot \vec{B}$$



$$\vec{F} = -\,\mathrm{grad}\left(U\right) \;=\; \mathrm{grad}\left(\vec{m} \cdot \vec{B}\right)$$

Dipolo magnetico: spira circolare

$$d\vec{B} = \frac{\mu_0}{4\pi} I\, ds \frac{\vec{u}_t \times \vec{u}_r}{r^2}$$



$$dB_x = dB\cos\theta = \frac{\mu_0}{4\pi}\frac{I\, ds}{r^2}\cos(\theta) =$$

$$= \frac{\mu_0}{4\pi} I\, ds \frac{1}{x^2 + R^2}\frac{R}{\sqrt{x^2 + R^2}}$$

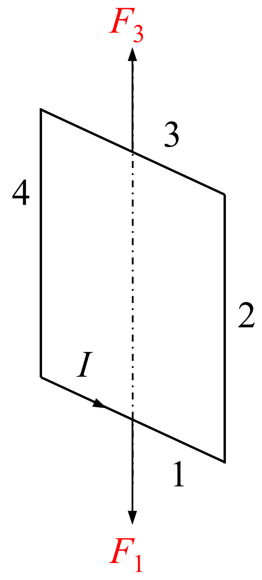
$$B = \int dB_x = \frac{\mu_0}{2}\frac{IR^2}{\left(x^2 + R^2\right)^{3/2}}$$

$$x \gg R: \quad B \approx \frac{\mu_0}{2}\frac{IR^2}{x^3} = \frac{\mu_0}{4\pi}\frac{2 \cdot I\pi R^2}{x^3} = \frac{\mu_0}{4\pi}\frac{2m}{x^3}$$

$$m = IS = I\pi R^2 \quad \Rightarrow \quad \vec{m} = I\vec{S}$$

momento di dipolo magnetico

Dipolo magnetico: spira rettangolare

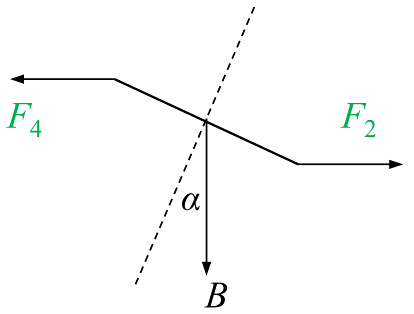


$$\vec{F} = I\int d\vec{s}\times\vec{B} = I\,\vec{s}\times\vec{B}$$

$$\begin{cases} F_1 = F_3 = Id_1 B\sin\left(\dfrac{\pi}{2} - \alpha\right) \\ F_2 = F_4 = Id_2 B \end{cases} \Rightarrow F_{tot} = 0$$

$$S = d_1 d_2$$

$$M = d_1 F_2 \sin(\alpha) = d_1 I d_2 B\sin(\alpha) = IS\cdot B\sin(\alpha) = mB\sin(\alpha)$$

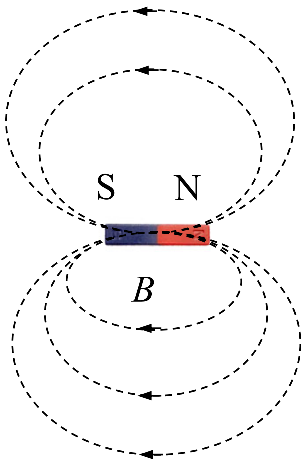


$$\vec{M} = \vec{m}\times\vec{B}$$

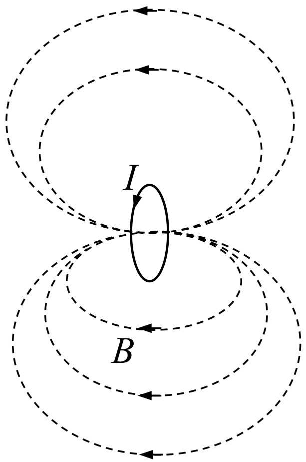
Teorema di equivalenza di Ampère

"Il campo magnetico creato e le interazioni subite
da un magnete e da una spira (in approx. di dipolo)
sono equivalenti"

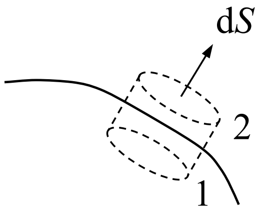
magnete



corrente



Formulazione differenziale: condizioni al contorno



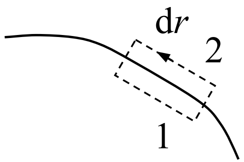
$$\begin{cases} d\Phi_2\left(\vec{B}\right) = \vec{B}_2 \cdot d\vec{S}_2 = B_{n2} dS \\ d\Phi_1\left(\vec{B}\right) = \vec{B}_1 \cdot d\vec{S}_1 = -\vec{B}_1 \cdot d\vec{S}_2 = -B_{n1} dS \\ d\Phi_{lat}\left(\vec{B}\right) \approx 0 \end{cases}$$

$$d\Phi\left(\vec{B}\right) = d\Phi_1\left(\vec{B}\right) + d\Phi_2\left(\vec{B}\right) + d\Phi_{lat}\left(\vec{B}\right) = \left(B_{n2} - B_{n1}\right) dS = \Delta B_n dS$$

$$d\Phi\left(\vec{B}\right) = 0$$

$$\Delta B_n = 0$$

Formulazione differenziale: condizioni al contorno



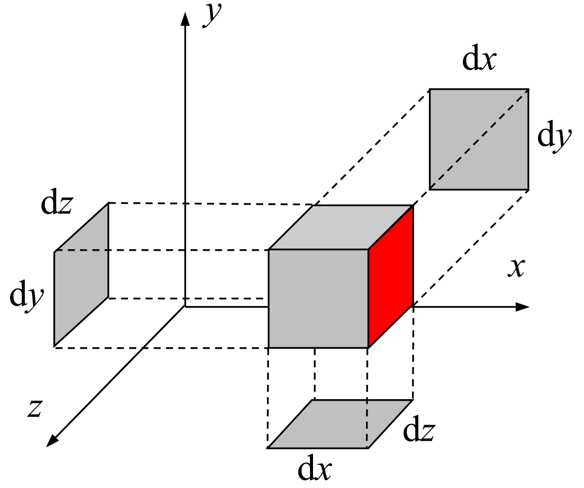
$$\begin{cases} d\Lambda_2\left(\vec{B}\right) = \vec{B}_2 \cdot d\vec{r}_2 = B_{t2}dr \\ d\Lambda_1\left(\vec{B}\right) = \vec{B}_1 \cdot d\vec{r}_1 = -\vec{B}_1 \cdot d\vec{r}_2 = -B_{t1}dr \\ d\Lambda_n\left(\vec{B}\right) \approx 0 \end{cases}$$

$$d\Lambda\left(\vec{B}\right) = d\Lambda_1\left(\vec{B}\right) + d\Lambda_2\left(\vec{B}\right) + d\Lambda_n\left(\vec{B}\right) = \left(B_{t2} - B_{t1}\right)dr = \Delta B_t dr$$

$$d\Lambda\left(\vec{B}\right) = \mu_0 dI = \mu_0 K_b dr$$

$$\Delta B_t = \mu_0 K_b$$

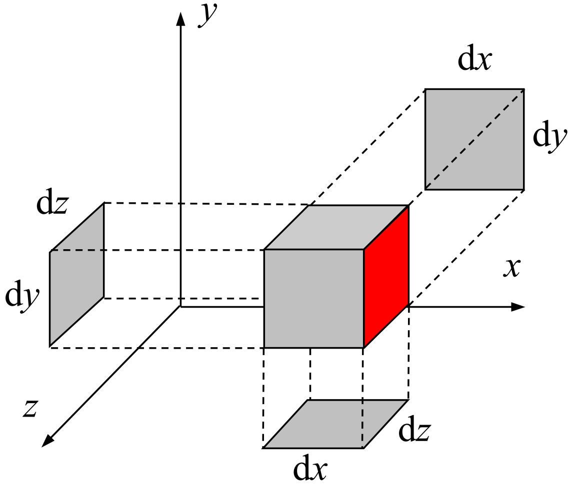
Formulazione differenziale: leggi di Maxwell



$$\begin{cases} d\Phi''_x\left(\vec{B}\right) = \vec{B}'' \cdot d\vec{S} = B''_x dS = B''_x dydz \\ d\Phi'_x\left(\vec{B}\right) = \vec{B}'_x \cdot d\vec{S} = -B'_x dS = -B'_x dydz \end{cases}$$

$$\begin{cases} d\Phi_x\left(\vec{B}\right) = d\Phi''_x\left(\vec{B}\right) + d\Phi'_x\left(\vec{B}\right) = B''_x dydz - B'_x dydz = dB_x dydz = \left(\dfrac{\partial B_x}{\partial x} dx\right) dydz = \dfrac{\partial B_x}{\partial x} dV \\ d\Phi_y\left(\vec{B}\right) = \dfrac{\partial B_y}{\partial y} dV \\ d\Phi_z\left(\vec{B}\right) = \dfrac{\partial B_z}{\partial z} dV \end{cases}$$

Formulazione differenziale: leggi di Maxwell

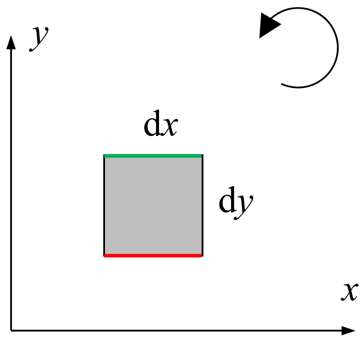


$$\mathrm{d}\Phi\left(\vec{B}\right) = \mathrm{d}\Phi_x\left(\vec{B}\right) + \mathrm{d}\Phi_y\left(\vec{B}\right) + \mathrm{d}\Phi_z\left(\vec{B}\right) = \left(\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}\right)\mathrm{d}V = \mathrm{div}\left(\vec{B}\right)\mathrm{d}V$$

$$\mathrm{d}\Phi\left(\vec{B}\right) = 0$$

$$\mathrm{div}\left(\vec{B}\right) = 0$$

Formulazione differenziale: leggi di Maxwell

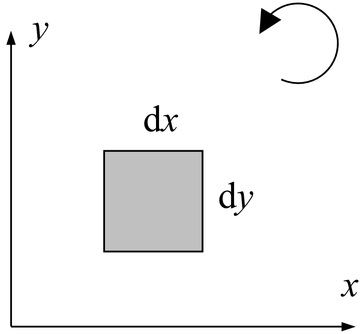


$$\begin{cases} d\Lambda_x'(\vec{B}) = \vec{B}' \cdot d\vec{r} = B_x' dx \\ d\Lambda_x''(\vec{B}) = \vec{B}'' \cdot d\vec{r} = -B_x'' dx \end{cases}$$

$$\begin{cases} d\Lambda_x(\vec{B}) = d\Lambda_x'(\vec{B}) + d\Lambda_x''(\vec{B}) = B_x' dx - B_x'' dx = -dB_x dx = -\left(\frac{\partial B_x}{\partial y} dy\right) dx \\ d\Lambda_y(\vec{B}) = dB_y dy = \frac{\partial B_y}{\partial x} dx dy \end{cases}$$

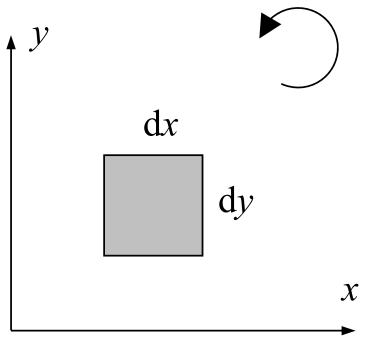
$$d\Lambda(\vec{B}) = d\Lambda_y(\vec{B}) + d\Lambda_x(\vec{B}) = \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) dx dy$$

Formulazione differenziale: leggi di Maxwell



$$\begin{cases} \text{piano } xy: & d\Lambda\left(\vec{B}\right) = \left(\dfrac{\partial B_y}{\partial x} - \dfrac{\partial B_x}{\partial y}\right) dx dy = \left(\dfrac{\partial B_y}{\partial x} - \dfrac{\partial B_x}{\partial y}\right) dS_z \\ \text{piano } yz: & d\Lambda\left(\vec{B}\right) = \left(\dfrac{\partial B_z}{\partial y} - \dfrac{\partial B_y}{\partial z}\right) dy dz = \left(\dfrac{\partial B_z}{\partial y} - \dfrac{\partial B_y}{\partial z}\right) dS_x \\ \text{piano } zx: & d\Lambda\left(\vec{B}\right) = \left(\dfrac{\partial B_x}{\partial z} - \dfrac{\partial B_z}{\partial x}\right) dz dx = \left(\dfrac{\partial B_x}{\partial z} - \dfrac{\partial B_z}{\partial x}\right) dS_y \end{cases}$$

Formulazione differenziale: leggi di Maxwell



$$d\Lambda\left(\vec{B}\right) = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right)dS_x + \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right)dS_y + \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)dS_z = \mathrm{rot}\left(\vec{B}\right)\cdot d\vec{S}$$

$$d\Lambda\left(\vec{B}\right) = \mu_0 dI = \mu_0 \vec{J}\cdot d\vec{S}$$

$$\mathrm{rot}\left(\vec{B}\right) = \mu_0 \vec{J}$$

Formulazione differenziale: leggi di Maxwell

	Teorema di Gauss	Teorema di Ampère
relazioni integrali	$\Phi\left(\vec{B}\right) = \oint \vec{B}\cdot \mathrm{d}\vec{S} = 0$	$\Lambda\left(\vec{B}\right) = \oint \vec{B}\cdot \mathrm{d}\vec{r} = \mu_0 I_c$
condizioni al contorno	$\Delta B_n = 0$	$\Delta B_t = \mu_0 K_b$
relazioni infinitesime	$\mathrm{div}\left(\vec{B}\right) = 0$	$\mathrm{rot}\left(\vec{B}\right) = \mu_0 \vec{J}$

Formulazione differenziale: identità vettoriali

$$
\begin{cases}
\mathrm{grad}(V) = \dfrac{\partial V}{\partial x}\vec{u}_x + \dfrac{\partial V}{\partial y}\vec{u}_y + \dfrac{\partial V}{\partial z}\vec{u}_z \\[2ex]
\mathrm{div}(\vec{E}) = \dfrac{\partial E_x}{\partial x} + \dfrac{\partial E_y}{\partial y} + \dfrac{\partial E_z}{\partial z} \\[2ex]
\mathrm{rot}(\vec{E}) = \begin{vmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}
\end{cases}
$$

derivate prime

$$\mathrm{div}(\vec{A}\times\vec{B}) = \mathrm{rot}(\vec{A})\cdot\vec{B} - \vec{A}\cdot\mathrm{rot}(\vec{B})$$

$$\mathrm{div}(f\vec{A}) = f\,\mathrm{div}(\vec{A}) + \vec{A}\cdot\mathrm{grad}(f)$$

$$
\begin{cases}
\mathrm{rot}(\mathrm{grad}(f)) \equiv 0 \\[2ex]
\mathrm{div}(\mathrm{rot}(\vec{A})) \equiv 0 \\[2ex]
\nabla^2(\vec{A}) \equiv \mathrm{grad}(\mathrm{div}(\vec{A})) - \mathrm{rot}(\mathrm{rot}(\vec{A}))
\end{cases}
$$

derivate seconde

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

teorema di Schwar(t)z

Formulazione differenziale: leggi di Maxwell

$$\begin{cases} \operatorname{div}\left(\vec{B}\right) = 0 \\ \operatorname{rot}\left(\vec{B}\right) = \mu_0 \vec{J} \end{cases}$$

$$\operatorname{div}\left(\operatorname{rot}\left(\vec{v}\right)\right) \equiv 0 \quad \Rightarrow \quad \vec{B} = \operatorname{rot}\left(\vec{A}\right)$$

$$\vec{A} = ?$$

$$\vec{B} = \int \mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}\int \operatorname{rot}\left(\frac{\vec{J}}{r}\right)\mathrm{d}V = \operatorname{rot}\left(\frac{\mu_0}{4\pi}\int \frac{\vec{J}}{r}\mathrm{d}V\right)$$

$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi} I\,\mathrm{d}s \frac{\vec{u}_t \times \vec{u}_r}{r^2} = \frac{\mu_0}{4\pi} JS\,\mathrm{d}s \frac{\vec{u}_t \times \vec{u}_r}{r^2} =$$

$$= \frac{\mu_0}{4\pi} J\mathrm{d}V \frac{\vec{u}_t \times \vec{u}_r}{r^2} = \frac{\mu_0}{4\pi}\mathrm{d}V \frac{\vec{J} \times \vec{u}_r}{r^2} =$$

$$= \frac{\mu_0}{4\pi}\mathrm{d}V \operatorname{rot}\left(\frac{\vec{J}}{r}\right)$$

potenziale vettore

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}}{r}\mathrm{d}V$$

vale la sovrapposizione degli effetti

Formulazione differenziale: leggi di Maxwell

$$\begin{cases} \mathrm{div}\left(\vec{B}\right) = 0 \\ \mathrm{rot}\left(\vec{B}\right) = \mu_0 \vec{J} \end{cases}$$

$$\mathrm{div}\left(\mathrm{rot}\left(\vec{v}\right)\right) \equiv 0 \quad \Rightarrow \quad \vec{B} = \mathrm{rot}\left(\vec{A}\right)$$

arbitraria

$$\vec{A} = \vec{A}_0 + \mathrm{grad}\left(f\right)$$

$$\mathrm{rot}\left(\vec{A}\right) = \mathrm{rot}\left(\vec{A}_0 + \mathrm{grad}\left(f\right)\right) =$$

$$= \mathrm{rot}\left(\vec{A}_0\right) + \cancel{\mathrm{rot}\left(\mathrm{grad}\left(f\right)\right)} = \mathrm{rot}\left(\vec{A}_0\right)$$

$$\mathrm{rot}\left(\vec{B}\right) = \mathrm{rot}\left(\mathrm{rot}\left(\vec{A}\right)\right) =$$

$$= \mathrm{grad}\left(\cancel{\mathrm{div}\left(\vec{A}\right)}\right) - \nabla^2\left(\vec{A}\right) = -\nabla^2\left(\vec{A}\right) = \mu_0 \vec{J}$$

gauge di Coulomb

equazione di Poisson