



POLITECNICO
MILANO 1863

Onde

Acustica. Onde elettromagnetiche. Ottica

Maurizio Zani

Onde

[Onde](#)

[Onde meccaniche](#)

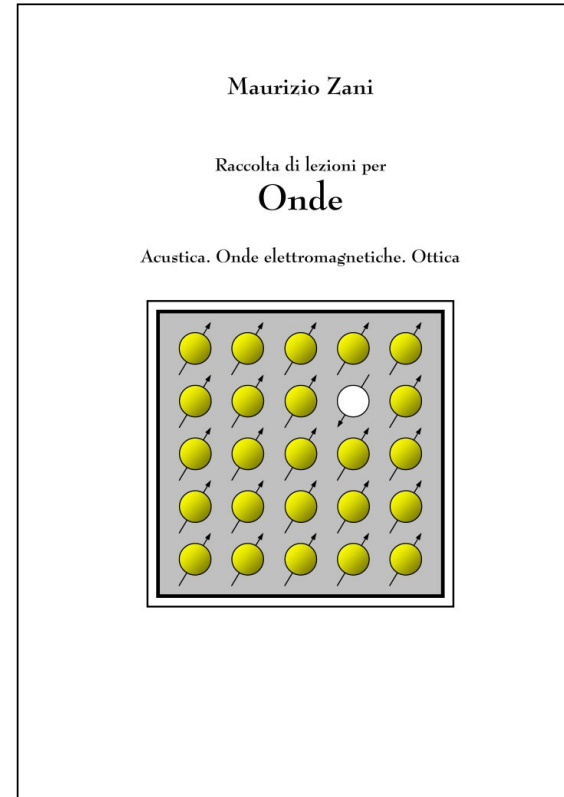
[Onde elettromagnetiche](#)

[Emissione e interazione elettromagnetica](#)

[Optica geometrica](#)

[Optica ondulatoria](#)

[Optica quantistica](#)



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Onde

Onde

Onde meccaniche

Onde elettromagnetiche

Emissione e interazione elettromagnetica

Ottica geometrica

Ottica ondulatoria

Ottica quantistica

Onde elettromagnetiche

Onde elettromagnetiche piane

Onde elettromagnetiche piane armoniche

Onde sferiche



emissione



propagazione



interazione

- superficie
- volume
- aperture/ostacoli

Onde elettromagnetiche

- velocità
- indice di rifrazione

Onde elettromagnetiche piane

- direzione
- ampiezze

Onde elettromagnetiche piane armoniche

- energia e intensità
- quantità di moto
- polarizzazione

senza sorgenti

$$\left\{ \begin{array}{l} \operatorname{div}(\vec{E}) = 0 \\ \operatorname{div}(\vec{B}) = 0 \\ \operatorname{rot}(\vec{E}) = -\frac{\partial \vec{B}}{\partial t} \\ \operatorname{rot}(\vec{B}) = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \end{array} \right.$$



eq. delle onde elettromagnetiche

$$\left\{ \begin{array}{l} \frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \\ \frac{\partial^2 \vec{B}}{\partial x^2} + \frac{\partial^2 \vec{B}}{\partial y^2} + \frac{\partial^2 \vec{B}}{\partial z^2} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0 \end{array} \right.$$

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$$

velocità di propagazione

$$\nabla^2(\vec{E}) = \operatorname{grad}(\operatorname{div}(\vec{E})) - \operatorname{rot}(\operatorname{rot}(\vec{E}))$$

nel vuoto

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c$$

in un mezzo

$$v = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}}$$

eq. delle onde elettromagnetiche

$$\begin{cases} \frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \\ \frac{\partial^2 \vec{B}}{\partial x^2} + \frac{\partial^2 \vec{B}}{\partial y^2} + \frac{\partial^2 \vec{B}}{\partial z^2} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0 \end{cases}$$

indice di rifrazione

$$n = \frac{c}{v} = \frac{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}}{\sqrt{\mu_0 \epsilon_0}} = \sqrt{\mu_r \epsilon_r} \approx \sqrt{\epsilon_r}$$



Onde elettromagnetiche piane: direzione e ampiezze

onda omogenea in xy

$$\begin{cases} \vec{E} = \vec{E}(z - ct) \\ \vec{B} = \vec{B}(z - ct) \end{cases} \quad \Rightarrow \quad \frac{\partial \vec{E}}{\partial x} = \frac{\partial \vec{E}}{\partial y} = \frac{\partial \vec{B}}{\partial x} = \frac{\partial \vec{B}}{\partial y} = 0$$

$$\text{div}(\vec{E}) = \cancel{\frac{\partial E_x}{\partial x}} + \cancel{\frac{\partial E_y}{\partial y}} + \cancel{\frac{\partial E_z}{\partial z}} = 0$$

$$\text{rot}(\vec{B})\Big|_z = \cancel{\frac{\partial B_y}{\partial x}} - \cancel{\frac{\partial B_x}{\partial y}} = \mu_0 \epsilon_0 \cancel{\frac{\partial E_z}{\partial t}}$$

$$\cancel{E_z; B_z} \quad \begin{cases} \vec{E} = (E_x; E_y) \\ \vec{B} = (B_x; B_y) \end{cases}$$



Onde elettromagnetiche piane: direzione e ampiezze

$$\left\{ \begin{array}{l} E_x = E_x(z - ct) \\ E_y = E_y(z - ct) \\ B_x = B_x(z - ct) \\ B_y = B_y(z - ct) \end{array} \right.$$

$$\frac{\partial \vec{E}}{\partial x} = \frac{\partial \vec{E}}{\partial y} = \frac{\partial \vec{B}}{\partial x} = \frac{\partial \vec{B}}{\partial y} = 0$$

$$\text{rot}(\vec{E})|_x = \cancel{\frac{\partial E_z}{\partial y}} - \frac{\partial E_y}{\partial z} = -\frac{\partial B_x}{\partial t}$$

$$\text{rot}(\vec{B})|_x = \cancel{\frac{\partial B_z}{\partial y}} - \frac{\partial B_y}{\partial z} = \mu_0 \epsilon_0 \frac{\partial E_x}{\partial t}$$

$$\left\{ \begin{array}{l} \frac{\partial E_y}{\partial z} = \frac{\partial B_x}{\partial t} \\ \frac{\partial B_y}{\partial z} = -\mu_0 \epsilon_0 \frac{\partial E_x}{\partial t} \end{array} \right.$$

$$w = z - ct$$

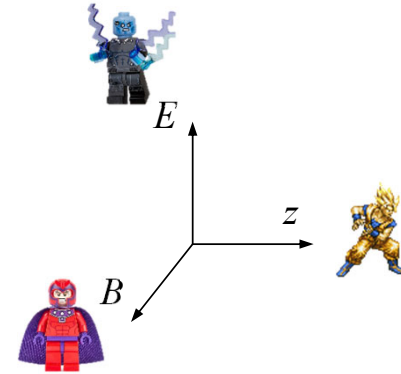
$$\left\{ \begin{array}{l} \frac{\partial E_y}{\partial w} \frac{\partial w}{\partial z} = \frac{\partial B_x}{\partial w} \frac{\partial w}{\partial t} \\ \frac{\partial B_y}{\partial w} \frac{\partial w}{\partial z} = \underbrace{-\mu_0 \epsilon_0}_{-1/c^2} \frac{\partial E_x}{\partial w} \frac{\partial w}{\partial t} \end{array} \right.$$

↗ 1 ↗ -c
↗ 1 ↗ -c



Onde elettromagnetiche piane: direzione e ampiezze

$$\left\{ \begin{array}{l} \frac{\partial E_y}{\partial w} = -c \frac{\partial B_x}{\partial w} \\ \frac{\partial B_y}{\partial w} = \frac{1}{c} \frac{\partial E_x}{\partial w} \end{array} \right. \quad \left\{ \begin{array}{l} E_y = -cB_x \\ E_x = cB_y \end{array} \right.$$



$$\vec{E} \cdot \vec{B} = E_x B_x + E_y B_y = cB_y B_x - cB_x B_y = 0$$

$$\vec{E} \perp \vec{B}$$

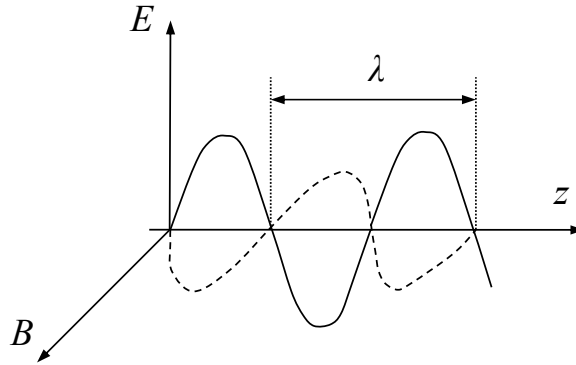
$$\vec{u}_E \times \vec{u}_B = \vec{u}_v$$

$$\frac{E}{B} = \frac{|\vec{E}|}{|\vec{B}|} = \frac{\sqrt{E_x^2 + E_y^2}}{\sqrt{B_x^2 + B_y^2}} = \frac{\sqrt{c^2 B_y^2 + c^2 B_x^2}}{\sqrt{B_x^2 + B_y^2}} = c$$

$$E = cB$$



Onde elettromagnetiche piane armoniche



$$\begin{cases} E = E_0 \sin[k(z - ct)] = E_0 \sin(kz - \omega t) \\ B = B_0 \sin[k(z - ct)] = B_0 \sin(kz - \omega t) \end{cases}$$

$$c = \frac{\omega}{k} = \frac{\lambda}{T}$$

pulsazione
lunghezza d'onda
periodo
numero d'onda



Onde elettromagnetiche piane armoniche: energia

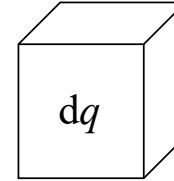
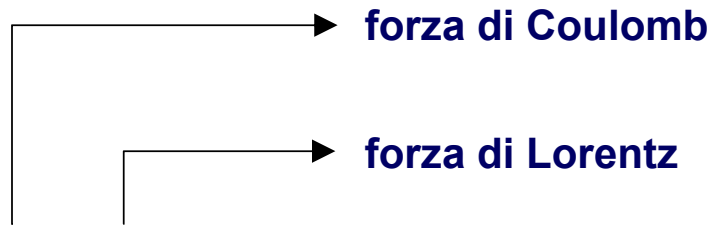
$$\left\{ \begin{array}{ll} \rho_e = \frac{dE_e}{dV} = \frac{1}{2} \varepsilon_0 E^2 & \text{densità di energia elettrica} \\ \rho_m = \frac{dE_m}{dV} = \frac{1}{2} \frac{1}{\mu_0} B^2 & \text{densità di energia magnetica} \end{array} \right. \quad \begin{array}{l} E = cB \\ c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \end{array}$$

$$\rho_{em} = \frac{dE_{em}}{dV} = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2} \frac{1}{\mu_0} B^2 = \varepsilon_0 E^2$$

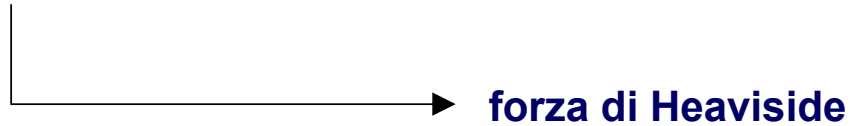
densità di energia elettromagnetica ?



Onde elettromagnetiche piane armoniche: energia



$$d\vec{F}_{tot} = d\vec{F}_e + d\vec{F}_m = dq \left(\vec{E} + \vec{v} \times \vec{B} \right)$$



$$dW = d\vec{F}_{tot} \cdot d\vec{r} = d\vec{F}_e \cdot d\vec{r} = dq \vec{E} \cdot \vec{v} dt = \vec{E} \cdot \vec{J} dV dt = \vec{E} \cdot \left(\frac{1}{\mu_0} \text{rot}(\vec{B}) - \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) dV dt$$

$$dW_m = 0$$

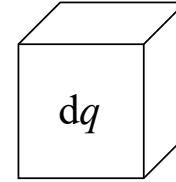
$$\vec{J} = nq_0 \vec{v} = \frac{dq}{dV} \vec{v}$$

$$\vec{J} = \frac{1}{\mu_0} \text{rot}(\vec{B}) - \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$



Onde elettromagnetiche piane armoniche: energia

$$\frac{dW}{dV dt} = \frac{dE_{mat}}{dV dt} = \vec{E} \cdot \frac{1}{\mu_0} \text{rot}(\vec{B}) - \varepsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$



$$\underbrace{\hspace{10em}} \quad \underbrace{-\varepsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} = -\frac{\partial}{\partial t} \left(\frac{1}{2} \varepsilon_0 E^2 \right)}$$

$$\frac{1}{\mu_0} \vec{E} \cdot \text{rot}(\vec{B}) = \frac{1}{\mu_0} \vec{B} \cdot \text{rot}(\vec{E}) - \frac{1}{\mu_0} \text{div}(\vec{E} \times \vec{B})$$

$$\underbrace{\hspace{10em}} \quad \frac{1}{\mu_0} \vec{B} \cdot \text{rot}(\vec{E}) = -\frac{1}{\mu_0} \vec{B} \cdot \frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t} \left(\frac{1}{2} \frac{1}{\mu_0} B^2 \right)$$



Onde elettromagnetiche piane armoniche: energia

$$\frac{dE_{mat}}{dV dt} = \vec{E} \cdot \frac{1}{\mu_0} \text{rot}(\vec{B}) - \epsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} = -\frac{\partial}{\partial t} \left(\frac{1}{2} \frac{1}{\mu_0} B^2 \right) - \frac{1}{\mu_0} \text{div}(\vec{E} \times \vec{B}) - \frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon_0 E^2 \right)$$



$$\frac{dE_{mat}}{dt} = -\frac{\partial}{\partial t} \int \left(\frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \frac{1}{\mu_0} B^2 \right) dV - \oint \frac{\vec{E} \times \vec{B}}{\mu_0} \cdot d\vec{S}$$

ρ_e ρ_m $\vec{S}_P = \frac{\vec{E} \times \vec{B}}{\mu_0}$

**vettore
di Poynting**

$$\oint \vec{S}_P \cdot d\vec{S} = - \left(\frac{dE_{mat}}{dt} + \frac{dE_{em}}{dt} \right)$$

teorema di Poynting



Onde elettromagnetiche piane armoniche: intensità

vettore di Poynting

$$\vec{S}_P = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

intensità

$$I = \left| \vec{S}_P \right| = \frac{EB}{\mu_0} = c\epsilon_0 E^2$$

onda armonica

$$E = E_0 \sin(kz - \omega t)$$

$$\langle S_P \rangle = c\epsilon_0 E_0^2 \langle \sin^2(kz - \omega t) \rangle = \frac{1}{2} c\epsilon_0 E_0^2$$

$$\langle I_0 \rangle = \langle S_P \rangle = \frac{1}{2} c\epsilon_0 E_0^2$$

nel vuoto

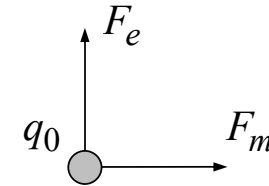
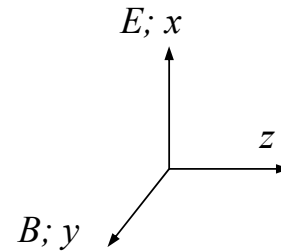
$$\langle I \rangle = \langle S_P \rangle = \frac{1}{2} v \epsilon E_0^2 = \frac{1}{2} \frac{c}{n} \epsilon_0 n^2 E_0^2 = n \langle I_0 \rangle$$

in un materiale



Onde elettromagnetiche piane armoniche: quantità di moto

$$\vec{F}_{tot} = \vec{F}_e + \vec{F}_m = q_0 (\vec{E} + \vec{v} \times \vec{B})$$



$$\left\{ \begin{aligned} W &= \int \vec{F}_e \cdot d\vec{r} = \int q_0 \vec{E} \cdot d\vec{r} = \int_0^T q_0 \vec{E} \cdot \vec{v} dt = \int_0^T q_0 E_x v_x dt = \Delta E \\ \Delta Q_z &= \int F_z dt = \int_0^T q_0 v_x B_y dt = \int_0^T q_0 v_x \frac{E_x}{c} dt = \frac{1}{c} \int_0^T q_0 v_x E_x dt = \frac{\Delta E}{c} \end{aligned} \right.$$

$$\Delta Q = \frac{\Delta E}{c}$$



Onde elettromagnetiche piane armoniche: quantità di moto

pressione di radiazione

$$p = \frac{1}{A} \frac{dQ}{dt} = \frac{1}{A} \frac{dE}{c dt} = \frac{1}{c} \frac{dE}{A dt} = \frac{I}{c}$$

$$\langle p \rangle = \frac{\langle I \rangle}{c} = \frac{1}{2} \epsilon_0 E_0^2$$

perfettamente
assorbente

$$p = \frac{I}{c}$$



onda e.m.

superficie



$$p = \frac{I}{c}$$

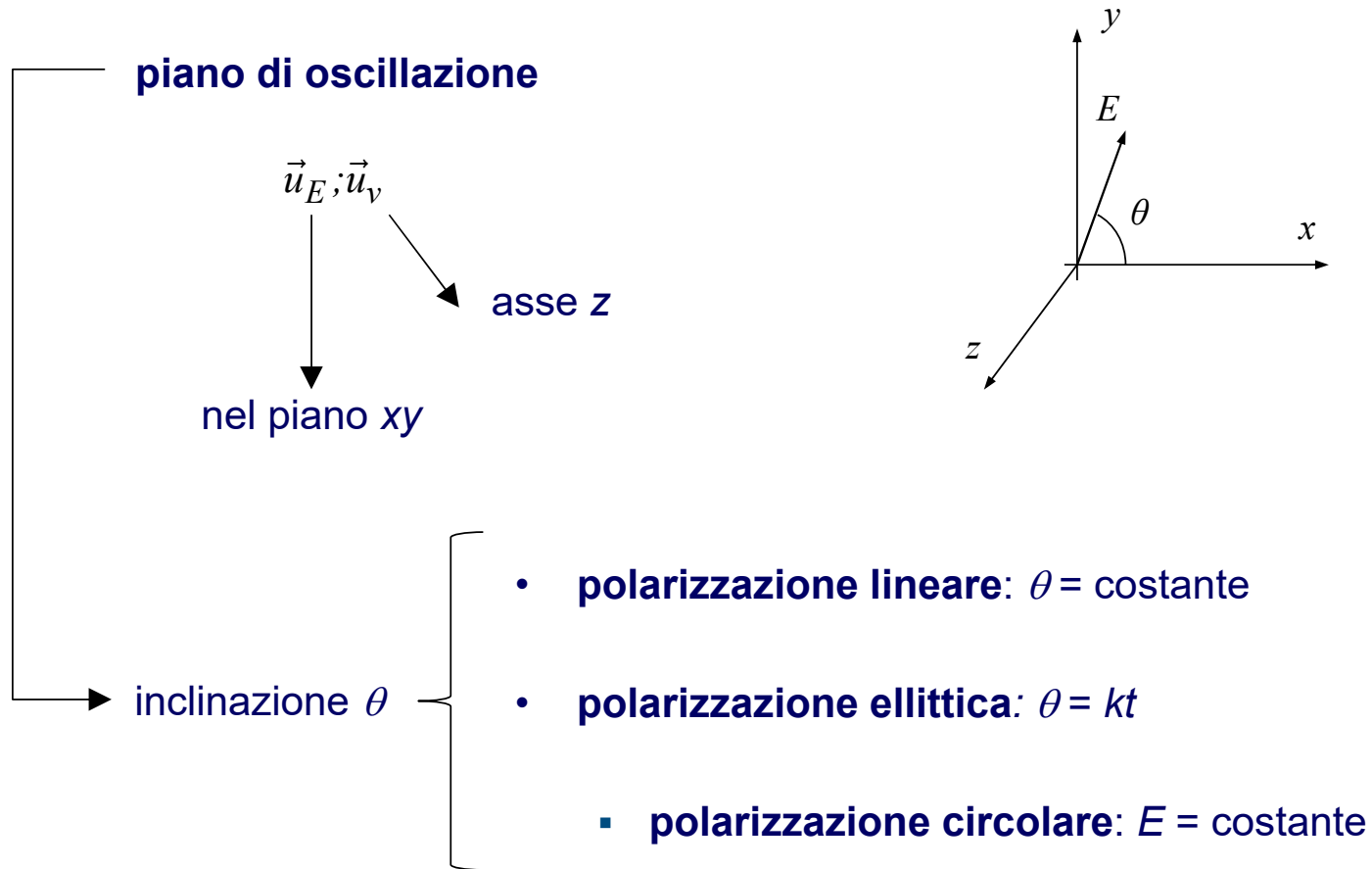
perfettamente
riflettente

$$p = \frac{I}{c}$$



$$p = 2 \frac{I}{c}$$

Onde elettromagnetiche piane armoniche: polarizzazione

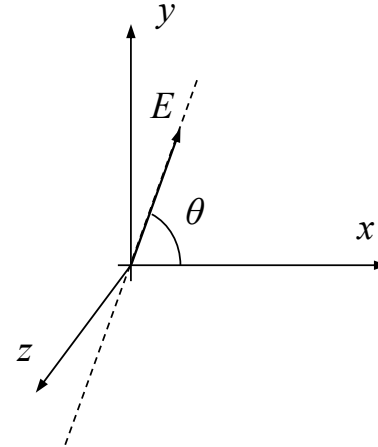


Onde elettromagnetiche piane armoniche: polarizzazione

polarizzazione lineare

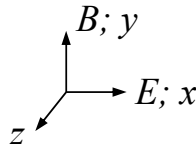
$$\vec{E} = E_{0x} \sin(kz - \omega t) \vec{u}_x + E_{0y} \sin(kz - \omega t) \vec{u}_y$$

$$\theta = \arctan\left(\frac{E_y}{E_x}\right) = \arctan\left(\frac{E_{0y}}{E_{0x}}\right)$$

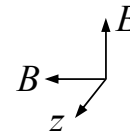


$$\text{se } \theta = 0 \quad \begin{cases} \vec{E} = E_0 \sin(kz - \omega t) \vec{u}_x \\ \vec{B} = B_0 \sin(kz - \omega t) \vec{u}_y \end{cases}$$

$$\text{se } \theta = \pi/2 \quad \begin{cases} \vec{E} = E_0 \sin(kz - \omega t) \vec{u}_y \\ \vec{B} = -B_0 \sin(kz - \omega t) \vec{u}_x \end{cases}$$



$$\vec{u}_E \times \vec{u}_B = \vec{u}_v$$

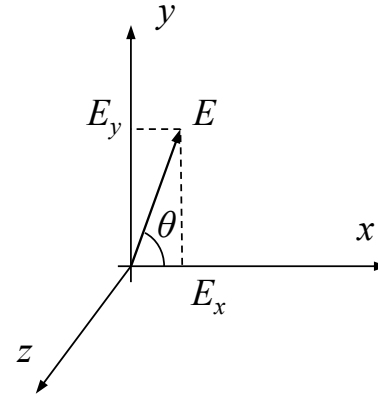


Onde elettromagnetiche piane armoniche: polarizzazione

combinazione di due onde
polarizzate linearmente & stessa pulsazione

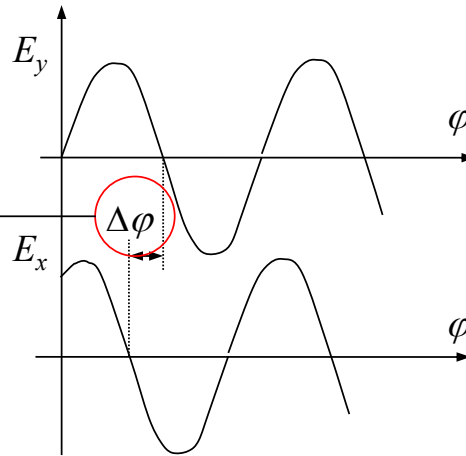
$$\vec{E} = E_{0x} \sin(kz - \omega t + \varphi_x) \vec{u}_x + E_{0y} \sin(kz - \omega t + \varphi_y) \vec{u}_y$$

$$\theta = \arctan\left(\frac{E_y}{E_x}\right)$$



$$\Delta\varphi = \varphi_y - \varphi_x$$

differenza di fase

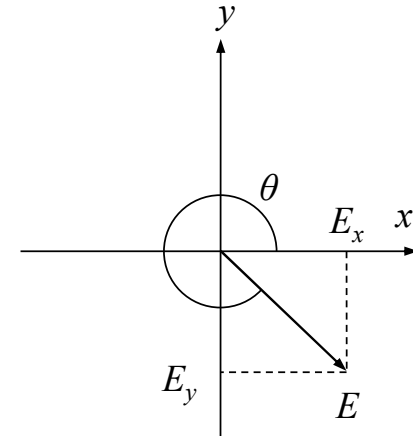
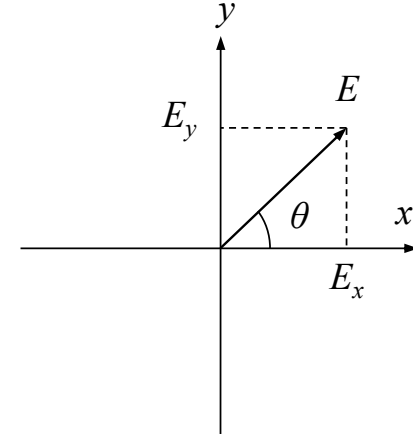
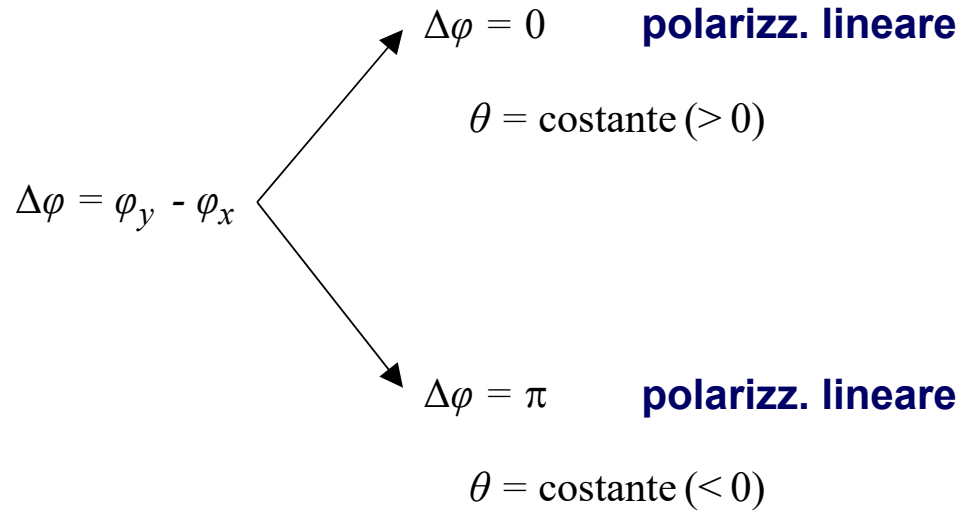


$$\frac{\Delta\varphi}{2\pi} = \frac{\Delta z}{\lambda} = \frac{\Delta t}{T}$$



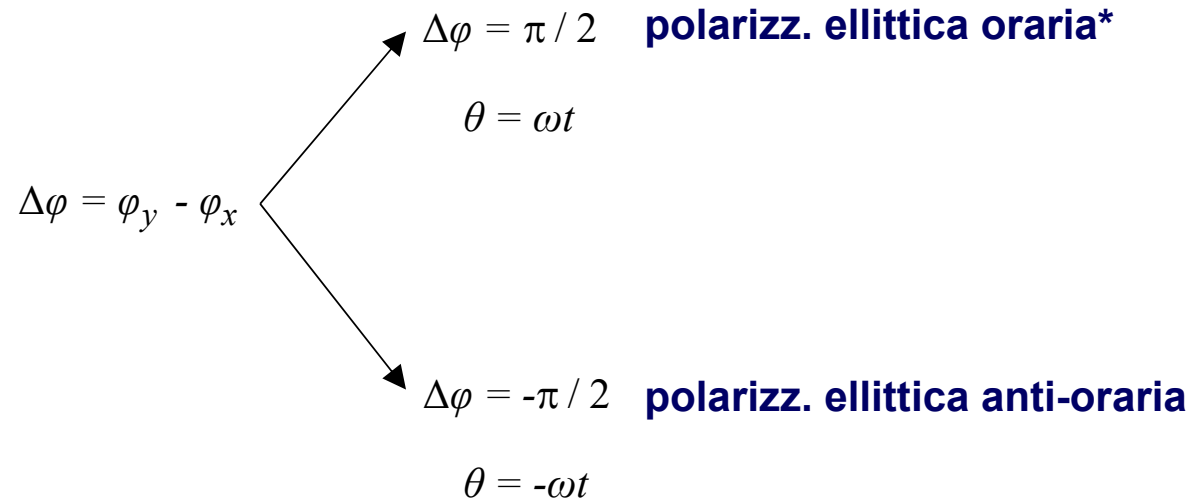
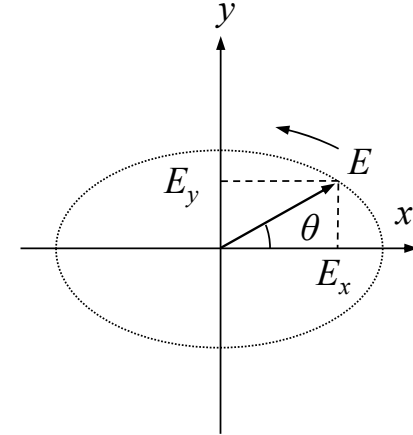
Onde elettromagnetiche piane armoniche: polarizzazione

$$\vec{E} = E_{0x} \sin(kz - \omega t + \varphi_x) \vec{u}_x + E_{0y} \sin(kz - \omega t + \varphi_y) \vec{u}_y$$



Onde elettromagnetiche piane armoniche: polarizzazione

$$\vec{E} = E_{0x} \sin(kz - \omega t + \varphi_x) \vec{u}_x + E_{0y} \sin(kz - \omega t + \varphi_y) \vec{u}_y$$

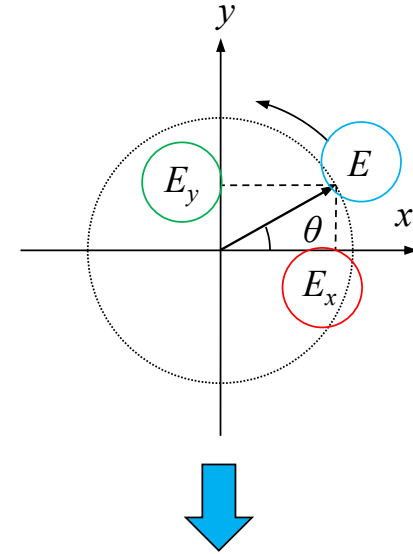
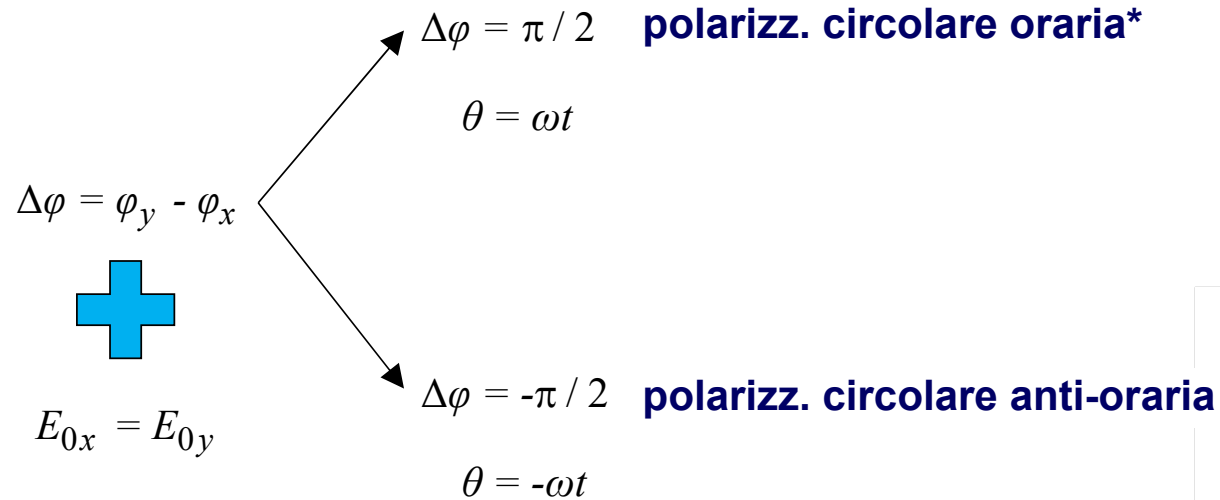


* o destra; onda osservata lungo il verso di propagazione



Onde elettromagnetiche piane armoniche: polarizzazione

$$\vec{E} = E_{0x} \sin(kz - \omega t + \varphi_x) \vec{u}_x + E_{0y} \sin(kz - \omega t + \varphi_y) \vec{u}_y$$



* o destra; onda osservata lungo il verso di propagazione

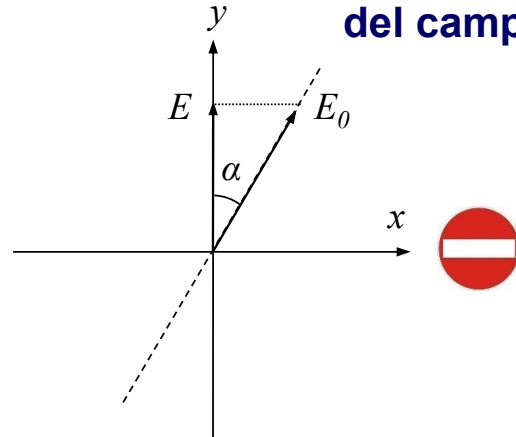


Onde elettromagnetiche piane armoniche: polarizzatore

asse del
polarizzatore



direzione
del campo E



polarizzazione lineare

$$\langle I_{in} \rangle = \frac{1}{2} c \epsilon_0 E_0^2$$

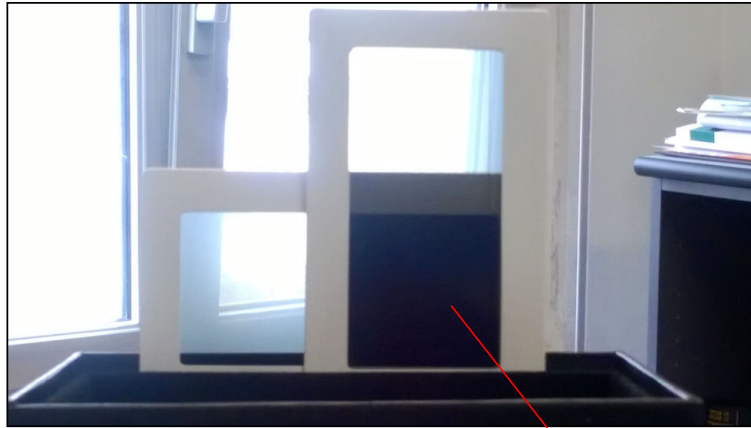
$$\langle I_{out} \rangle = \frac{1}{2} c \epsilon_0 E^2 = \frac{1}{2} c \epsilon_0 (E_0 \cos(\alpha))^2 = \langle I_{in} \rangle \cos^2(\alpha)$$

legge di Malus



Onde elettromagnetiche piane armoniche: polarizzatore

2 polarizzatori ortogonali



intensità nulla

2 polarizzatori ortogonali
con interposto 1 obliquo



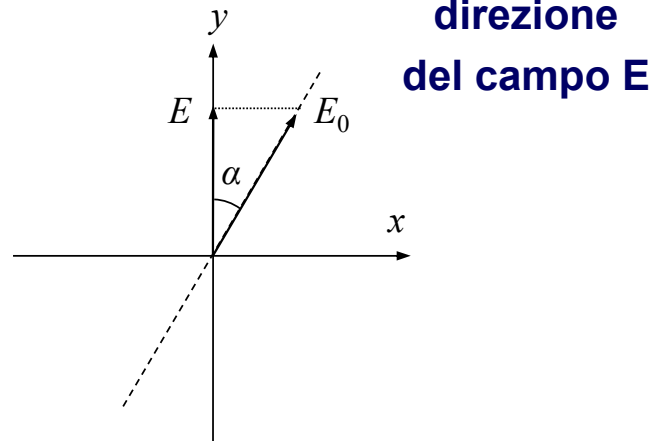
intensità nulla

intensità non nulla!



Onde elettromagnetiche piane armoniche: polarizzatore

asse
del polarizzatore



polarizzazione circolare

$$\langle I_{in} \rangle = \frac{1}{2} c \epsilon_0 E_0^2 = \langle I_{in x} \rangle + \langle I_{in y} \rangle = 2 \langle I_{in y} \rangle$$

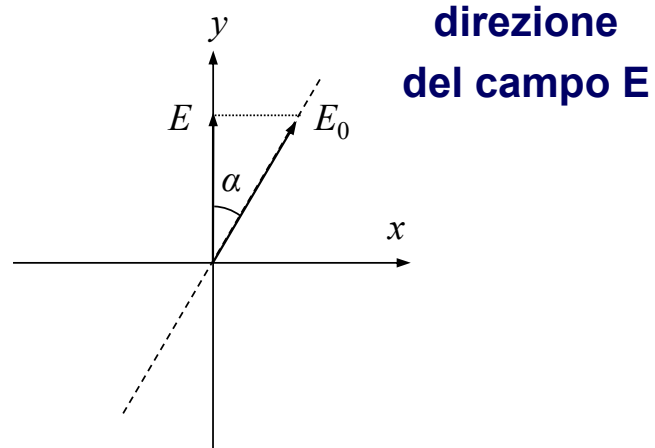
$$\langle I_{out} \rangle = \langle I_{in y} \rangle = \frac{1}{2} \langle I_{in} \rangle$$

temporalmente



Onde elettromagnetiche piane armoniche: polarizzatore

**asse
del polarizzatore**



luce non polarizzata

$$\langle I_{in} \rangle = \frac{1}{2} c \epsilon_0 E_0^2 = \langle I_{in x} \rangle + \langle I_{in y} \rangle = 2 \langle I_{in y} \rangle$$

$$\langle I_{out} \rangle = \langle I_{in y} \rangle = \frac{1}{2} \langle I_{in} \rangle$$

statisticamente

